Problem Set 3

ECON 30020, Intermediate Macroeconomics, Fall 2024
The University of Notre Dame
Professor Sims

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, you may turn in one problem set, but make sure that the names of all group members are clearly legible at the top of your assignment. Problem sets should be handed in during class and stapled in the upper left corner. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers / figures from Excel – you need not turn in Excel code. This problem set is due at the beginning of class on October 16.

1. **Precautionary Saving:** Suppose that you have a household that lives for two periods, t and $t+1$. Period-t income is known with certainty, as is the real interest rate on borrowing/saving. Future income is unknown. Future income may take on two values: $Y_{t+1}^h > Y_{t+1}^l$, with $p \in [0,1]$ being the probability of the "high" state and $1-p$ the probability of the "low" state.

   The household wishes to maximize expected lifetime utility, with a standard utility function and discount factor:

   $$\mathbb{E}[U] = u(C_t) + \beta \left[p u(C_{t+1}^h) + (1-p) u(C_{t+1}^l) \right]$$

   Subject to budget constraints:

   $$C_t + S_t = Y_t$$
   $$C_{t+1}^h = Y_{t+1}^h + (1+r_t) S_t$$
   $$C_{t+1}^l = Y_{t+1}^l + (1+r_t) S_t$$

   (a) Use calculus to derive a first-order optimality condition that implicitly characterizes optimal savings, S_t, as a function of things taken as given by the household.

   (b) Show that this first-order condition may be written as a standard consumption Euler equation, but with an expectations operator, $\mathbb{E}(\cdot)$, in front of expected future marginal utility.

   (c) Suppose that the utility function is the natural log, i.e. $u(C_t) = \ln C_t$. Suppose further that $Y_t = 1$, $r_t = 0.02$, and $\beta = (1+r_t)^{-1}$. Suppose that $Y_{t+1}^h = 1.1$ and $Y_{t+1}^l = 0.9$, with $p = 1/2$. Download the "Solver" add-on in Microsoft Excel (see **HERE** for more information on how to do this and how to use the tool). Numerically solve for optimal savings, S_t. Use this to derive a numeric optimal value of current consumption, C_t.

   (d) Suppose, instead, that $Y_{t+1}^h = 1.2$ and $Y_{t+1}^l = 0.8$. Verify that this does not affect the expected value of future income. Re-do (c). Comment on how optimal savings and consumption are impacted.

(e) Now, instead suppose that the utility function is $u(C_t) = C_t - \frac{aC_t^2}{2}$, where $a > 0$. Produce expressions for the first and second derivatives of this utility function. Provide a parameter restriction on a such that marginal utility is always positive. What is true about the sign of the second derivative?

(f) Re-do (c) and (d) with the utility function given in (e). Assume that a = 0.1. Solve for optimal savings and consumption for both the future income options in (c) and in (d). What happens to saving and consumption when future income becomes more uncertain? Can you say something about why?

2. **The MPC and IS Curve:** Write down the generic definition of the IS curve for an endowment economy. Graphically show to how derive the IS curve. How will the MPC affect the slope of the IS curve (i.e. if the MPC is bigger, will the IS curve be flatter or steeper)? For the purposes of this exercise, assume that the sensitivity of consumption to the interest rate is independent of the value of the MPC. Show graphically and briefly provide some written intuition.

3. **Precautionary Savings in General Equilibrium:** Re-do question (1) (parts (a) - (d), only), but instead of taking the interest rate as given, suppose that we have a representative agent economy with market-clearing condition $S_t = 0$ (equivalently, $Y_t = C_t$ and $Y_{t+1} = C_{t+1}$). Solve for the equilibrium real interest rate in both future income setups – how does the equilibrium interest rate change when the future becomes more uncertain? Draw a IS-Y^s curve and provide some written intuition for your answer.

4. **Heterogeneity, Transfers, and Equilibrium:** Suppose that you have an endowment economy populated by two agents – labeled (1) and (2). Each type of agent has log utility, an identical discount factor, and faces the same interest rate. But they face potentially different endowment patterns. Accordingly, the consumption function for each type of agent (j = (1) or (2)) is:

$$C_t(j) = \frac{1}{1+\beta}\left[Y_t(j) + \frac{Y_{t+1}(j)}{1+r_t}\right]$$

Type 1 agents have endowment pattern $(Y_t(1), Y_{t+1}(1)) = (0, 1)$. Type 2 agents have endowment pattern $(Y_t(2), Y_{t+1}(2)) = (1, 0)$. Suppose there are 100 of each type of agent.

(a) Market-clearing requires that aggregate savings equal 0, or, equivalently, that $C_t = 100 \times C_t(1) + 100 \times C_t(2) = Y_t = 100 \times Y_t(1) + 100 \times Y_t(2)$. Solve for an expression for the equilibrium r_t using the given endowment patterns.

(b) Given this answer, find expressions for the equilibrium amount of savings of each type of agent, e.g., $S_t(1)$ and $S_t(2)$. In equilibrium, which agent is borrowing, and which is saving? And what is true about aggregate savings?

(c) Argue that the equilibrium r_t you found in part (a) is the same as if there were just one type of agent with the same preferences and endowment pattern $(Y_t, Y_{t+1}) = (100, 100)$.

(d) Suppose that a benevolent social planner implements a tax and transfer scheme to equalize the first-period endowments across agents. That is, the social planner taxes type 2 agents 1/2 and gives 1/2 as a transfer payment to type 1 agents (since there are an equal number of agents, this is "budget neutral" from the perspective of the social planner). Re-do part (a) under this assumption. How does the tax and transfer system impact the equilibrium interest rate? Re-do part (b). How does the tax and transfer system influence the individual saving/borrowing of each type of agent?