Equilibrium with Production and Endogenous Labor Supply

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Chapter 12

Production and Labor Supply

We continue working with a two period, optimizing, equilibrium model of the economy

No uncertainty over future, although it would be straightforward to entertain this

Model augmented along the following two dimensions:

- We model production and an investment decision
- Model endogenous labor supply

The production side is very similar to the Solow model

Firm

There exists a representative firm. The firm produces output using capital, K_t, and labor, N_t, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

A_t is exogenous productivity variable. Abstract from trend growth

$F(\cdot)$ has the same properties as assumed in the Solow model – increasing in both arguments, concave in both arguments, both inputs necessary. For example:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

Capital Accumulation

Slightly differently than the Solow model, we assume that the firm makes the capital accumulation decisions

We assume that the firm must borrow from a financial intermediary in order to finance its investment

"Equity" versus "debt" finance would be equivalent absent financial frictions, which we will model

Furthermore, ownership of capital wouldn't make a difference absent financial frictions (i.e. firm makes capital accumulation decision vs. household owning capital and leasing it to firms)

Current capital, K_t, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

Prices Relevant for the Firm

Firm hires labor in a competitive market at (real) wage w_t (and w_{t+1} in the future)

Firm borrows to finance investment at

$$r_t^I = r_t + f_t$$

r_t^I is the interest rate relevant for the firm, while r_t is the interest rate relevant for the household

f_t is (an exogenous) variable representing a financial friction. We will refer to this as a credit spread

During financial crises observed credit spreads rise significantly

Dividend

The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend

Dividend is simply output less cost of labor in period t (since borrowing cost of investment is borne in future)

$$D_t = Y_t - w_t N_t$$

Future Dividend and Terminal Condition

Terminal condition for the firm: firm wants $K_{t+2} = 0$ (die with no capital). This implies $I_{t+1} = -(1-\delta)K_{t+1}$, which we can think of as the firm "liquidating" its remaining capital after production in $t+1$

This is an additional source of revenue for the firm in $t+1$. In addition, firm has to pay interest plus principal on its borrowing for investment in t:

$$D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1} - (1+r_t^I)I_t$$

Firm Valuation

Value of the firm: PDV of flow of dividends:

$$V_t = D_t + \frac{1}{1 + r_t} D_{t+1}$$

The relevant interest rate for discounting future profit is r_t, not r_t^l

This is because household earns the firm and discounts future dividend

Firm Problem

Firm problem is to pick N_t and I_t to maximize V_t subject to accumulation equation:

$$\max_{N_t, I_t} \; V_t = D_t + \frac{1}{1+r_t} D_{t+1}$$

s.t.

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$D_t = A_t F(K_t, N_t) - w_t N_t$$

$$D_{t+1} = A_{t+1} F(K_{t+1}, N_{t+1}) + (1-\delta)K_{t+1} - w_{t+1} N_{t+1} - (1 + r_t^I) I_t$$

First-Order Conditions

$$w_t = A_t F_N(K_t, N_t)$$

$$1 + r_t^I = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta)$$

Intuition: MB = MC

Wage condition exactly same as Solow model expression for wage

Investment condition can be re-written in terms of earlier notation by noting $R_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$ and:

$$R_{t+1} = r_t^I + \delta = r_t + f_t + \delta$$

Return on capital, R_{t+1}, closely related to real interest rate, r_t

Diversion: Debt vs. Equity Finance

We are assuming firm finances investment via debt. Equity finance:

$$D_t = Y_t - w_t N_t - I_t$$

$$D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1}$$

Debt: lower dividend in future. Equity: lower dividend in present

FOC w/ equity:

$$1 + r_t = A_{t+1}F_K(K_{t+1}, N_{t+1}) + (1-\delta)$$

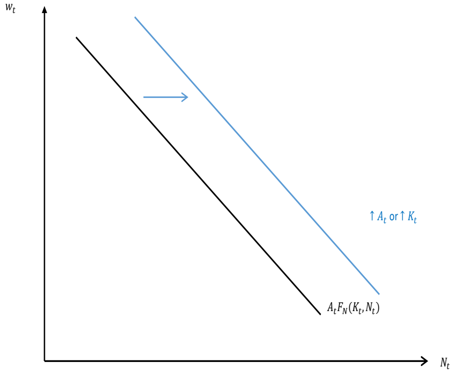
Firm would prefer equity if $f_t > 0$; otherwise, firm is indifferent (Modigliani-Miller 1958)

We assume some underlying friction prevents firm's ability to issue debt

Labor Demand

Labor FOC implicitly characterizes a downward-sloping labor demand curve:

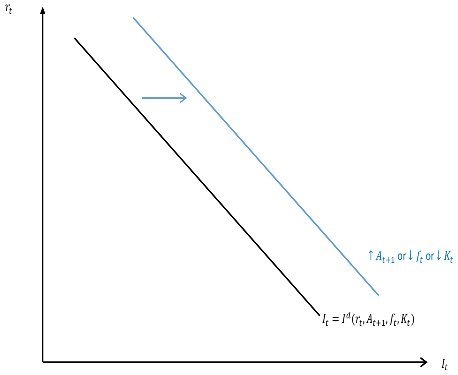
$$N_t = N^d(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t})$$



Investment Demand

Second first order condition implicitly defines a demand for K_{t+1}, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I^d(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{f_t}, \underset{-}{K_t})$$



Household

There exists a representative household. Households gets utility from consumption and <u>leisure</u>, where leisure is $L_t = 1 - N_t$, with N_t labor and available time normalized to 1

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

Example flow utility functions:

$$u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$$
$$u(C_t, 1 - N_t) = \ln \left[C_t + \theta_t \ln(1 - N_t) \right]$$

Here, θ_t is an exogenous "labor supply shock" governing utility from leisure (equivalently, disutility from labor)

Notation: u_C denotes marginal utility of consumption, u_L marginal utility of leisure (marginal utility of labor is $-u_L$)

Budget Constraints

Household faces two flow budget constraints, conceptually the same as before, but now income is partly endogenous:

$$C_t + S_t \leq w_t N_t + D_t$$
$$C_{t+1} + S_{t+1} - S_t \leq w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1} + r_t S_t$$

Household takes D_t, D_{t+1}, and D^I_{t+1} (dividend from financial intermediary) as given (ownership different than management)

Terminal condition: $S_{t+1} = 0$. Gives rise to IBC:

$$C_t + \frac{C_{t+1}}{1 + r_t} = w_t N_t + D_t + \frac{w_{t+1} N_{t+1} + D_{t+1} + D^I_{t+1}}{1 + r_t}$$

First-Order Conditions

Do the optimization in the usual way. The following first order conditions emerge:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_{t+1})$$

This is the usual Euler equation, only looks different to accommodate utility from leisure

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$
$$u_L(C_{t+1}, 1 - N_{t+1}) = w_{t+1} u_C(C_{t+1}, 1 - N_{t+1})$$

Discussion and intuition

Optimal Decision Rules

Can go from first order conditions to optimal decision rules

Cutting a few corners, we get the same consumption function as before:

$$C_t = C^d(\underset{+}{Y_t}, \underset{+}{Y_{t+1}}, \underset{-}{r_t})$$

Or, if there were government spending, with Ricardian Equivalence we'd have:

$$C_t = C^d(Y_t \underset{+}{-} G_t, Y_{t+1} \underset{+}{-} G_{t+1}, \underset{-}{r_t})$$

Labor Supply

First-order condition for N_t can be characterized by an indifference curve / budget line diagram similar to the two period consumption case

Things are complicated for a few reasons:

- Competing income and substitution effects of w_t
- Non-wage income and expectations about future income (including through an interest rate channel) can affect current labor supply

Labor Supply with GHH Preferences

Labor supply can actually be quite complicated

We will sweep most of this stuff under rug: no income effects and other things (other than exogenous variable θ_t) are ignored

Can be motivated explicitly with preference specification due to Greenwood, Hercowitz, and Huffman (1988):

$$u(C_t, 1 - N_t) = \ln\left[C_t + \theta_t \ln(1 - N_t)\right]$$

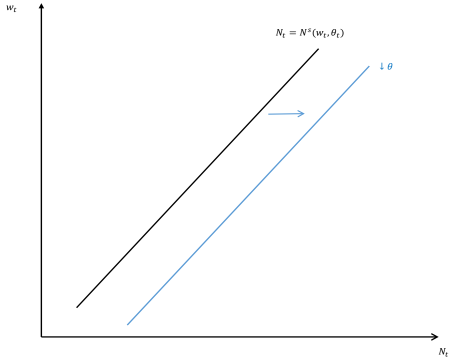
FOC becomes:

$$\frac{\theta_t}{1 - N_t} = w_t$$

Labor Supply Curve

Labor supply function under these assumptions:

$$N_t = N^s(\underset{+}{w_t}, \underset{-}{\theta_t})$$



Financial Intermediary

Will not go into great detail

In period t, takes in deposits, S_t, from household; issues loans in amount I_t to firm

Pays r_t for deposits, and earns $r_t^I = r_t + f_t$ on loans

f_t is exogenous, and $f_t > 0$ means intermediary earns profit in $t+1$, which is returned to household as dividend:

$$D_{t+1}^I = (r_t + f_t)I_t - r_t S_t$$

Market-Clearing

Market-clearing requires $S_t = I_t$ (i.e. funds taken in by financial intermediary equal funds distributed to firm for investment)

This implies:

$$Y_t = C_t + I_t$$

If there were a government levying (lump sum) taxes on household period t resource constraint would just be:

$$Y_t = C_t + I_t + G_t$$

Equilibrium

The following conditions must all hold in period t in equilibrium:

$$
\begin{aligned}
C_t &= C^d(Y_t, Y_{t+1}, r_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
I_t &= I^d(r_t, A_{t+1}, f_t, K_t) \\
Y_t &= A_t F(K_t, N_t) \\
Y_t &= C_t + I_t
\end{aligned}
$$

Endogenous: C_t, N_t, Y_t, I_t, w_t, and r_t

Exogenous: A_t, A_{t+1}, K_t, f_t, θ_t. Will talk about Y_{t+1} and K_{t+1} later

Four optimal decision rules, two resource constraints: income = production and income = expenditure

Competitive Equilibrium

There are now <u>two</u> prices – r_t (intertemporal price of goods) and w_t (price of labor)

Different ways to think about what the markets are. One is clear – market for labor, which w_t adjusts to clear (i.e. labor supply = demand)

Can think about either market for goods (i.e. $Y_t = C_t + I_t$) or a loanable funds market $S_t = I_t$ as being the other market, which r_t adjusts to clear. We will focus on market for goods

Endowment economy special case of this if N_t and I_t are held fixed

Will be possible to do some consumption smoothing in equilibrium here, however. Suppose household wants to increase S_t. It can do this if r_t falls to incentivize more I_t (whereas in endowment economy $I_t = 0$, so S_t must remain fixed at 0).