# Midterm 2 Review

ECON 30020: Intermediate Macroeconomics
Professor Sims
University of Notre Dame, Fall 2024

The second midterm will take place on Monday, November 11. In terms of the order of coverage, the material relevant for the exam includes the following chapters from GLS: Chapter 11 (competitive equilibrium in an endowment economy), Chapter 13 (fiscal policy and Ricardian Equivalence), Chapter 12 (equilibrium with endogenous production), Chapters 18-19 (graphical analysis of the neoclassical model), Chapter 14 (money demand). The exam will consistent of 10 multiple choice questions (for which no justification is required), 10 true/false questions (for which no justification is required), and several "free response" questions which require either a written response in words, algebraic calculations, or graphical analysis. The free response questions ought to look something like the problem set questions. You may use a calculator (though you should not need one), but you may not access notes, phones, or anything else during the exam. This review lists some terms you should know, asks some basic questions, includes some new problems for practice, and references problems from GLS. It is meant to help you study; it is not a substitute for reviewing the text, lecture notes, and problem sets.

I posted two old exams – one from Fall 2016, one from Spring 2018. The course coverage is not identical across semesters. In studying for this exam, you can omit the following questions from each past exam:

- Fall 2016
  - Nothing. Everything on this exam is fair game.
- Spring 2018
  - Multiple choice #7
  - Multiple choice #8
  - Multiple choice #9
  - True/false #5
  - True/false #6
  - True/false #7
  - True/false #8
  - Free-response question #3 (we probably won't get to the coverage of how to determine the values of endogenous nominal variables, but the rest of the problem is fair game)

I start by listing some key variables / parameters that often come up:

- $Y_t$: output produced at time $t$
- $C_t, I_t$: consumption and investment at time $t$
- $K_t$: capital stock
- $N_t$: labor input (hours) at time $t$
- $L_t$: leisure. The time endowment is normalized to 1, so $L_t = 1 - N_t$
- $w_t$: real wage (denominated in units of goods)
- $r_t$: real interest rate
- $D_t$: dividend paid out by firm in period $t$
- $V_t$: value of the firm (PDV of dividends)
- $T_t$: tax obligation of household in period $t$
- $G_t$: government spending in period $t$
- $S_t$: the real stock of savings (e.g. bonds) which the household takes from period $t$ into $t+1$
- $B_t$: the real stock of debt issued by the government which it takes from $t$ to $t+1$
- $\alpha$: exponent on capital in a Cobb-Douglas production function
- $\delta$: depreciation rate on capital
- $\beta$: discount factor
- $\theta_t$: exogenous variable that impacts the disutility from supplying labor (equivalently, the preference for leisure)
- $A_t$: current productivity (exogenous)
- $A_{t+1}$: future productivity (exogenous)
- $f_t$: credit spread shock (exogenous)
- $r_t^I$: real interest rate relevant for firm, equal to $r_t^I = r_t + f_t$
- $D_{t+1}^I$: real dividend household receives from ownership in financial intermediary
- $M_t$: the stock of money (exogenously set by the government)
- $P_t$: the price of goods measured in units of money (i.e. dollars per good)
- $\frac{M_t}{P_t}$: real money balances

- $i_t$: the nominal interest rate

- $\pi^e_{t+1}$: expected inflation (rate of growth in the price level) from $t$ to $t+1$. Taken to be exogenous.

- $\pi_t$: inflation rate, i.e. $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

Next, I list key expressions/formulae that frequently come up:

- $Y_t = C_t + I_t + G_t$ (different expenditure categories; for much of what we do there is no government and no rest of world, so $G_t = 0$). This is sometimes called the aggregate resource constraint. It comes from imposing asset market-clearing (total saving equals total investment) and the definition of dividends in the household's period-$t$ budget constraint.

- $K_{t+1} = I_t + (1-\delta)K_t$: capital accumulation equation. $K_t$ is predetermined and exogenous within a period (it depends on past investment decisions).

- $Y_t = A_t F(K_t, N_t)$: production function in the neoclassical model

- $U = u(C_t, 1 - N_t) + v\left(\frac{M_t}{P_t}\right) + \beta u(C_{t+1}, 1 - N_{t+1})$: most general representation of lifetime utility for the household. If there is no endogenous labor choice just eliminate the dependence of $u(\cdot)$ on $1 - N_t$. If there is no money in the model just eliminate the function $v(\cdot)$.

- $u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_{t+1})$: consumption Euler equation. If not endogenous labor just eliminate the dependence on $1 - N_t$ and you have the Euler equation we derived earlier.

- $u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$: first order optimality condition for labor supply. Has interpretation of marginal benefit equals marginal supply

- $1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$: Fisher relationship relating real and nominal interest rates. In approximate form (take logs), you have $r_t = i_t - \pi^e_{t+1}$.

- $v'\left(\frac{M_t}{P_t}\right) = \frac{i_t}{1+i_t} u_C(C_t, 1 - N_t)$: first order optimality condition for money holdings. Has the interpretation of marginal benefit equals marginal cost.

- $w_t = A_t F_N(K_t, N_t)$: first order condition for optimal labor demand

- $1 + r^I_t = A_{t+1}F_K(K_{t+1}, N_{t+1}) + (1 - \delta)$: first order condition implicitly characterizing optimal investment demand

- $C_t + S_t = Y_t - T_t$ and $C_{t+1} = Y_{t+1} - T_{t+1} + (1 + r_t)S_t$: the period $t$ and $t+1$ flow budget constraints in a model where income is exogenous (i.e. an endowment economy) (imposing the terminal condition and assuming these hold with equality)

- $C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$: intertemporal budget constraint (IBC) in the endowment economy

- $G_t \leq T_t + B_t$: government's period $t$ flow budget constraint in a real model with no money
- $G_{t+1} + (1+r_t)B_t \leq T_{t+1}$: government's period $t+1$ flow budget constraint in real model with no money
- $G_t + \frac{G_{t+1}}{1+r_t} = T_t + \frac{T_{t+1}}{1+r_t}$: government's intertemporal budget constraint in a real model with no money
- $C_t + S_t = w_t N_t - T_t + D_t$ and $C_{t+1} = w_{t+1}N_{t+1} - T_{t+1} + (1+r_t)S_t + D_{t+1}$: period $t$ and $t+1$ budget constraints in a real economy with endogenous labor supply and production
- $C_t + \frac{C_{t+1}}{1+r_t} = w_t N_t - T_t + D_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + D_{t+1}}{1+r_t}$: intertemporal budget constraint in an economy with endogenous production and labor supply
- $D_t = Y_t - w_t N_t$: firm dividend in period $t$
- $D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1} - (1+r_t^I)I_t$: firm dividend in period $t+1$
- $V_t = D_t + \frac{D_{t+1}}{1+r_t}$: value of the firm
- $C_t + S_t + \frac{M_t}{P_t} \leq w_t N_t - T_t + D_t$ and $C_{t+1} = w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + (1+r_t)S_t + \frac{M_t}{P_{t+1}}$: period $t$ and $t+1$ budget constraints (written in real terms) in an economy with money
- $C_t + \frac{C_{t+1}}{1+r_t} + \frac{i_t}{1+i_t}\frac{M_t}{P_t} = w_t N_t - T_t + D_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + D_{t+1}}{1+r_t}$: intertemporal budget constraint in an economy with endogenous production, labor supply, and money
- $G_t \leq T_t + B_t + \frac{M_t}{P_t}$ and $G_{t+1} + (1+i_t)\frac{P_t}{P_{t+1}}B_t + \frac{M_t}{P_{t+1}} \leq T_{t+1}$: real flow budget constraints for the government when there is money
- $G_t + \frac{G_{t+1}}{1+r_t} = T_t + \frac{T_{t+1}}{1+r_t} + \frac{i_t}{1+i_t}\frac{M_t}{P_t}$: real intertemporal budget constraint for the government when there is money
- The equilibrium conditions of the full neoclassical model with money:

$$\begin{aligned}
C_t &= C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t) \\
N_t &= N^s(w_t, \theta_t) \\
N_t &= N^d(w_t, A_t, K_t) \\
I_t &= I^d(r_t, f_t, A_{t+1}, K_t) \\
Y_t &= A_t F(K_t, N_t) \\
Y_t &= C_t + I_t + G_t \\
M_t &= P_t M^d(i_t, Y_t) \\
r_t &= i_t - \pi_{t+1}^e
\end{aligned}$$

If working in a real economy with no money, then just omit the last two equations.

- Equilibrium values of real endogenous variables ($Y_t$, $r_t$, $N_t$, and $w_t$ can be graphically found in the following five-part graph. You may have to actually think to figure out $C_t$ and $I_t$. You don't need to know anything about nominal variables (endogenous or exogenous) to determine the real endogenous variables in the neoclassical model:

Next are some questions for review.

1. Provide brief definitions (or, in some cases, a brief discussion) for the following terms:

   (i) Precautionary saving

   (ii) Desired aggregate expenditure

   (iii) Autonomous expenditure

   (iv) $IS$ curve

   (v) Income and substitution effect for the effects of a wage change on labor supply

   (vi) Competitive equilibrium

   (vii) Lump sum taxes

   (viii) Ricardian equivalence

   (ix) The fiscal multiplier for a fixed real interest rate (with and without Ricardian equivalence)

   (x) Dividends

   (xi) Fisher relationship

   (xii) Fiat money

   (xiii) Double coincidence of wants

   (xiv) $Y^s$ curve

   (xv) Modigiliani-Miller theorem

   (xvi) Liquidation value of firm

   (xvii) Classical dichotomy

   (xviii) Monetary neutrality

   (xix) Crowding out

   (xx) Demand shock, supply shock

2. Suppose that we have an endowment economy, but with two different types of agents. There are sufficiently many of each type of agent that they all behave as price-takers. The agents differ in their endowment streams – type 1 agents have endowment pattern $(Y_t^1, Y_{t+1}^1) = (1, 0)$, while type 2 agents have endowment pattern $(Y_t^2, Y_{t+1}^2) = (0, 1)$. In words, type 1 agents have income today but none in the future, while type 2 agents have no income today but one unit in the future. Let there be $N^1$ of type 1 agents and $N^2$ of type 2 agents. These agents can save borrow or borrow at the common real interest rate, $r_t$.

(a) Before doing any math, do you think that saving/borrowing will take place in equilibrium in this economy? If so, what type of agents will be saving? Which type will be borrowing? Why?

The problem of the type 1 agents is:

$$\max_{C_t^1, S_t^1, C_{t+1}^1} \quad U = \ln C_t^1 + \beta \ln C_{t+1}^1$$

s.t.

$$C_t^1 + S_t^1 = 1$$
$$C_{t+1}^1 = (1 + r_t) S_t^1$$

(b) Combine the two within period constraints into one intertemporal budget constraint and derive the Euler equation characterizing an optimal consumption plan for type 1 agents.

(c) Derive the consumption function for type 1 agents.

(d) Use your consumption function from (b) to derive a saving function for type 1 agents (e.g. $S_t^1 = Y_t^1 - C_t^1$, so just plug in your consumption function).

The problem of type 2 agents is:

$$\max_{C_t^2, S_t^2, C_{t+1}^2} \quad U = \ln C_t^2 + \beta \ln C_{t+1}^2$$

s.t.

$$C_t^2 + S_t^2 = 0$$
$$C_{t+1}^2 = 1 + (1 + r_t) S_t^2$$

(e) Combine the two within period constraints into one intertemporal budget constraint and derive the Euler equation characterizing an optimal consumption plan for type 2 agents.

(f) Derive the consumption function for type 2 agents.

(g) Use your consumption function from (b) to derive a saving function for type 2 agents (e.g. $S_t^2 = Y_t^2 - C_t^2$, so just plug in your consumption function).

(h) In equilibrium, what must be true about $N^1 S_t^1$ (aggregate saving of type 1 agents) and $N^2 S_t^2$ (aggregate saving of type 2 agents)?

(i) Use the equilibrium condition from (h) to solve for the equilibrium real interest rate, as well as the equilibrium consumption allocations of each type of agent (e.g. $C_t^1$ and $C_t^2$).

(j) Suppose that there is an increase in the number of type 2 agents (e.g. $N^2$ increases). How will this affect the equilibrium real interest rate and the consumption allocations? Will type 1 agents be better or worse off following the increase in the population of type 2 agents?

3. The IS curve is conceptually the same in a production economy as in an endowment economy, but in a production economy there is an additional expenditure category (investment). If the consumption function is exactly the same in both economies, will the IS curve be steeper, flatter, or the same slope in the production economy as in the endowment economy? Show with graphs and explain in words why.

4. For the production economy, graphically derive the $IS$ curve.

5. For the production economy, graphically derive the $Y^s$ curve.

6. After an increase in $A_t$, will the $Y^s$ curve shift more or less if the labor supply curve is more or less elastic (i.e. flat)? Will a change in $A_t$ have a bigger or smaller effect on $r_t$ in equilibrium if the labor supply curve is more elastic (i.e relatively flat)? Explain.

7. Evaluate and discuss the following statement. “Provided agents can freely borrow/lend with one another, the distribution of endowments (conditional on the aggregate endowments) is irrelevant for the equilibrium real interest.”

8. What does it mean to treat $Y_{t+1}$ as being “pseudo-exogenous” in the neoclassical model?

9. Use graphs and other analysis to reproduce on pg. 26 of the slides titled “money_micro_slides_sp2018.pdf.”

10. A neoclassical model with a twist. Suppose that the equations characterizing the model are the same as above, with the exception that labor supply depends on the real interest rate. In particular, we have:

$$N_t = N^s(w_t, \theta_t, r_t)$$

It is assumed that labor supply is increasing in the real interest rate.

(a) Graphically derive the $Y^s$ curve with this alternative labor supply curve.

(b) Can you say with certainty how $N_t$ would react to an increase in $A_t$ in this version of the model? Explain briefly.

(c) Suppose that there is an increase in $G_t$. Show how this effects the equilibrium of the model. Is the government spending multiplier in equilibrium zero, positive but less than one, equal to one, or greater than one? Explain.

(d) Do $C_t$ and $I_t$ react more or less to an increase in $G_t$ when labor supply is a function of the real interest rate in comparison to our standard case?

11. Suppose that there is a firm with a *fixed* amount of physical capital, $\bar{K}$ (no time subscript since it is fixed). Its only variable input is labor. In period $t$, the firm is forced to borrow to finance its payments to labor. It has to pay this back in period $t+1$. The firm does not have to borrow to finance payments to labor in period $t+1$. The firm's dividends are:

$$D_t = A_t \bar{K}^{\alpha} N_t^{1-\alpha}$$

$$D_{t+1} = A_{t+1} \bar{K}^{\alpha} N_{t+1}^{1-\alpha} - w_{t+1} N_{t+1} - (1 + r_t) w_t N_t$$

The value of the firm is:

$$V_t = D_t + \frac{D_{t+1}}{1 + r_t}$$

Derive the optimality condition for the firm's choice of $N_t$. Argue that it is not affected by $r_t$, in spite of the fact that the firm must borrow to finance its labor payment. What is the intuition for this?

12. List a couple of situations in which Ricardian Equivalence would *not* hold. Provide some reasoning for why.

13. Provide some intuition for the following statement: "The equilibrium real interest rate is a measure of how plentiful the future is expected to be relative to the present."

14. GLS Chapter 11, Exercises 1-2 and 4

15. GLS Chapter 12, Exercise 1

16. GLS Chapter 13, Questions 4-6

17. GLS Chapter 14, Exercise 1

18. GLS Chapter 14, Questions 3-7

19. GLS Chapter 18, Exercises 1-2

20. GLS Chapter 19, Exercises 1-6

21. GLS Chapter 19, Questions 2-3