Problem Set 4

ECON 30020, Intermediate Macroeconomics, Fall 2024
The University of Notre Dame
Professor Sims

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, you may turn in one problem set, but make sure that the names of all group members are clearly legible at the top of your assignment. Problem sets should be handed in during class and stapled in the upper left corner. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers / figures from Excel – you need not turn in Excel code. This problem set is due at the beginning of class on November 6.

1. Consider a firm solving a static profit optimization problem. Output is produced using the production technology $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. Capital is predetermined and you may think of it as fixed for this problem. The firm hires labor at real wage w_t, which the firm takes as given.

   (a) Write down the profit function (i.e., expression for the period-t dividend) for the firm.

   (b) Use calculus to derive a condition characterizing the optimal labor choice by the firm. Provide some written intuition for why this condition must hold.

   (c) Re-arrange the condition you found in (b) to isolate N_t on the left hand side – i.e., derive an expression for the labor demand curve.

   (d) Suppose that $\alpha = 1/3$, $K_t = 10$, and $A_t = 1$. In Excel, consider values of w_t ranging from 1.7 to 4.2 (with a "space" of 0.1 between). For each possible wage, solve for the optimal value of N_t. Produce a plot of the labor demand curve (with w_t on the vertical axis and N_t on the horizontal axis).

   (e) Re-do part (d), but this time with a value of $A_t = 1.1$. Graphically show how a higher A_t impacts the position of the labor demand curve.

2. Now, consider a dynamic value-maximization problem for a firm choosing how much to invest. The is endowed with an initial capital stock, K_t. It does not use labor. The production function is $Y_t = A_t K_t^{\alpha}$. Capital accumulates according to: $K_{t+1} = I_t + (1-\delta)K_t$. The firm must borrow to finance investment at (net) interest rate $r_t^I = r_t + f_t$. The value of the firm is the present discounted value of dividends, where future dividends are discounted by $1/(1+r_t)$.

   (a) Write down an expression for the value of the firm. Explain, in words, why this expression is the value of the firm.

   (b) Use calculus to derive a condition implicitly characterizing optimal investment by the firm. Provide some written intuition for why this condition must hold.

   (c) Re-arrange your answer from (b) to isolate investment on the left hand side – i.e., derive an expression for the investment demand curve.

   (d) Suppose that $\alpha = 1/3$, $K_t = 5$, $A_{t+1} = 1$, $\delta = 0.05$, and $f_t = 0.01$. Create a grid of possible values of r_t ranging from 0.01 up to 0.05 (with a space of 0.001 in between). For each r_t, solve for optimal investment, I_t. Plot the investment demand function from Excel with r_t on the vertical axis and I_t on the horizontal axis.

(e) Re-do part (d), but this time assume a value of $A_{t+1} = 1.1$. Graphically show how a higher A_{t+1} impacts the position of the investment demand curve.

(f) Re-do part (d) (this time again assuming $A_{t+1} = 1$, but now with $f_t = 0.015$. Graphically show how a higher f_t impacts the position of the investment demand curve.

3. **Understanding Labor Supply** Consider a static household model. The household gets utility from consumption, C_t, and leisure, $L_t = 1 - N_t$, where N_t is labor. The household earns income from labor, $w_t N_t$, taking the wage, w_t, as given. The household's problem is:

$$\max_{C_t, N_t} \quad u(C_t, 1 - N_t)$$

s.t.

$$C_t = w_t N_t$$

(a) For the general utility function, $u(\cdot)$, derive the first-order optimality condition for this problem. Provide some written intuition for why this condition must hold if the household is behaving optimally.

(b) Suppose that the utility function is $u(C_t, 1 - N_t) = \ln\left[C_t + \theta_t \ln(1 - N_t)\right]$, where θ_t is an exogenous preference shifter. Use your generic first-order condition from (a), along with this utility function, to derive a labor supply function for the household.

(c) Instead, suppose that the utility function is $u(C_t, 1 - N_t) = \ln C_t + \theta_t \ln(1 - N_t)$. Use the generic first-order condition from (a), along with this utility function, to derive a labor supply function for the household.

(d) Can you provide any economic intuition for the differences between the labor supply functions you find in (b) and (c)? Explain.

4. Suppose that you have consumption and investment demand functions with the usual properties:

$$C_t = C^d(Y_t, Y_{t+1}, r_t)$$
$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

Market-clearing requires that $Y_t = C_t + I_t$.

(a) Graphically derive the IS curve.

(b) Relative to the endowment economy world in which there is no investment, should the IS curve be steeper or flatter here? Explain briefly.

(c) Suppose that f_t increases. Graphically show how the IS curve shifts.

5. Suppose that we have a standard production function, i.e., $Y_t = A_t F(K_t, N_t)$ with the usual properties. Suppose that instead of our "typical" assumption that labor supply is a function solely of the wage and the preference variable θ_t, i.e., $N_t = N^s(w_t, \theta_t)$, labor supply is also a function of the real interest rate, $N_t = N^s(w_t, \theta_t, r_t)$. If $\frac{\partial N^s(w_t, \theta_t, r_t)}{\partial r_t} = 0$, we are in our "typical" case. Suppose, for this problem, that $\frac{N^s(w_t, \theta_t, r_t)}{\partial r_t} > 0$.

(a) Provide some brief, written intuition for why it might make sense to assume that $\frac{N^s(w_t,\theta_t,r_t)}{\partial r_t} > 0$.

(b) Graphically derive the Y^s curve when $\frac{N^s(w_t,\theta_t,r_t)}{\partial r_t} > 0$.

(c) Explain how and why the Y^s curve looks different when $\frac{N^s(w_t,\theta_t,r_t)}{\partial r_t} > 0$ than when $\frac{N^s(w_t,\theta_t,r_t)}{\partial r_t} = 0$.