Problem Set 5

ECON 30020, Intermediate Macroeconomics, Fall 2024
The University of Notre Dame
Professor Sims

Instructions: You may work on this problem set in groups of up to four people. Should you choose to do so, you may turn in one problem set, but make sure that the names of all group members are clearly legible at the top of your assignment. Problem sets should be handed in during class and stapled in the upper left corner. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers / figures from Excel – you need not turn in Excel code. This problem set is due at the beginning of class on December 9.

1. **Equilibrium Efficiency in the Neoclassical Model** Suppose that we have an economy populated by a representative household and a representative firm. There is no government. There is no money. Everything is real. There is no uncertainty. The household solves the following problem:

$$\max_{C_t, N_t, S_t} \quad U = \ln C_t + \theta_t \ln(1 - N_t) + \beta \left[\ln C_{t+1} + \theta_{t+1} \ln(1 - N_{t+1}) \right]$$

s.t.

$$C_t + S_t = w_t N_t + D_t$$

$$C_{t+1} = w_{t+1} N_{t+1} + D_{t+1} + (1 + r_t) S_t$$

D_t and D_{t+1} are dividends received from ownership in the firm; these are taken as given by the household.

The representative firm produces output according to $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$. Its dividends each period are:

$$D_t = Y_t - w_t N_t$$

$$D_{t+1} = Y_{t+1} - w_{t+1} N_{t+1} + (1 - \delta) K_{t+1} - (1 + r_t) I_t$$

The firm is required to borrow to finance all investment in period t. We assume that $f_t = 0$ (so the rate the government pays to borrow is the same as the rate the household earns on saving). Investment turns into new capital via:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

The firm wants to pick N_t and I_t to maximize the present discounted value of dividends subject to the law of motion for physical capital:

$$\max_{N_t, I_t} \quad V_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t + \frac{1}{1 + r_t} \left[A_{t+1} K_{t+1}^{\alpha} N_{t+1}^{1-\alpha} - w_{t+1} N_{t+1} + (1 - \delta) K_{t+1} - (1 + r_t) I_t \right]$$

(a) Use calculus to derive first-order optimality conditions for the household.

(b) Use calculus to derive first-order optimality conditions for the firm.

(c) Derive the aggregate market-clearing condition.

Now, consider a social planner's version of this problem. The social planner's aggregate resource constraint is $Y_t = C_t + I_t$, or $Y_t = C_t + K_{t+1} - (1-\delta)K_t$ in period t substituting out I_t in terms of K_t and K_{t+1}. In period $t+1$, the constraint is $Y_{t+1} = C_t - (1-\delta)K_{t+1}$ (leftover capital gets to be consumed). The social planner wants to pick C_t, N_t, and K_{t+1} (equivalently, I_t, given a value of K_t) to maximize the household's utility subject to these constraints:

$$\max_{C_t, N_t, K_{t+1}} \quad U = \ln C_t + \theta_t \ln(1-N_t) + \beta\left[\ln C_{t+1} + \theta_{t+1}\ln(1-N_{t+1})\right]$$

s.t.

$$C_t + K_{t+1} - (1-\delta)K_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$
$$C_{t+1} = A_{t+1}K_{t+1}^{\alpha}N_{t+1}^{1-\alpha} + (1-\delta)K_{t+1}$$

Note that there are no prices (r_t or w_t) in the social planner's problem.

(d) Use calculus to derive first-order optimality conditions for the planner's problem.

(e) Show that the first-order conditions for the competitive equilibrium (parts (a) - (c)) imply the same quantities as those from the social planner's problem. Hint: just eliminate prices in the competitive allocation.

2. **The Aggregate Demand Curve:** The equations characterizing the demand side of the New Keynesian model (and the neoclassical model, for that matter) are:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$
$$Y_t = C_t + I_t + G_t$$
$$\frac{M_t}{P_t} = M^d(i_t, Y_t)$$
$$r_t = i_t - \pi_{t+1}^e$$

(a) What are the exogenous variables in these equations and what are the endogenous variables? Please list them.

(b) Write down (in words) the definition of the IS curve. Which of the equations above are summarized by the IS curve?

(c) Write down (in words) the definition of the LM curve. Which of the equations above are summarized by the LM curve?

(d) Write down (in words) the definition of the AD curve.

(e) Suppose that, contrary to our standard assumptions, both consumption and investment are completely insensitive to the real interest rate (i.e. $\frac{\partial C^d(\cdot)}{\partial r_t} = \frac{\partial I^d(\cdot)}{\partial r_t} = 0$). What will the IS and AD curves look like under these assumptions? How do they look different relative to our standard model?

(f) Revert to assuming that consumption and investment are both decreasing in the real interest rate. Instead assume that money demand is insensitive to the nominal interest rate – i.e. $\frac{\partial M^d(\cdot)}{\partial i_t} = 0$. What will the LM and AD curves look like under this assumption? How do they look different relative to our standard model?

3. **Algebra in a Simple Sticky Price Model with no Investment or Government:** Suppose that we have an economy with no investment and no government. Output is produced using labor only. The price level is completely sticky. Assume the following specific functional forms for the relevant equations of the model:

$$C_t = aY_t + bY_{t+1} - cr_t$$
$$N_t = \theta_t w_t$$
$$Y_t = A_t N_t$$
$$P_t = \bar{P}_t$$
$$Y_t = C_t$$
$$\frac{M_t}{P_t} = -di_t + eY_t$$
$$r_t = i_t - \pi^e_{t+1}$$

The parameters a, b, c, d, and e are non-negative constants. The remaining variables have their usual interpretations.

(a) What are the exogenous variables and what are the endogenous variables? List them.

(b) Explain intuitively why we do not have a labor demand curve when the price level is sticky.

(c) Algebraically solve for an expression for the IS curve.

(d) Algebraically solve for an expression for the LM curve.

(e) Algebraically solve for an expression for the AD curve.

(f) Combine your algebraic expression for the AD curve with the given AS curve ($P_t = \bar{P}_t$) to express equilibrium output, Y_t as a function of exogenous variables only.

(g) How is the sensitivity of output to the money supply (i.e. $\frac{\partial Y_t}{\partial M_t}$), impacted by the sensitivity of consumption demand to the real interest rate (as measured by the parameter c)? Can you provide any intuition for this?

(h) Use your previous work to derive expression for how Y_t, N_t, and w_t are impacted by changes in A_t. Can you provide any intuition for your answers?

4. **An Interest Rate Peg:** Suppose that we have a partial sticky price model with one twist. The twist is this – the central bank desires to keep the nominal interest rate constant at some specified value, $i_t = \bar{i}$, where $\bar{i}$ is an exogenous target. This means that M_t becomes an *endogenous* variable rather than exogenous – in the face of exogenous shocks, the central bank must adjust M_t in such a way as to keep $i_t = \bar{i}$. The equations characterizing the model are standard with the exception of this additional equation and the fact that M_t is now endogenous rather than exogenous.

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$
$$P_t = \bar{P}_t + \gamma(Y_t - Y_t^f)$$
$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$
$$Y_t = A_t F(K_t, N_t)$$
$$Y_t = C_t + I_t + G_t$$
$$M_t = P_t M^d(i_t, Y_t)$$
$$r_t = i_t - \pi^e_{t+1}$$
$$i_t = \bar{i}$$

   (a) Briefly discuss how the parameter γ nests both the simple sticky price model and the neoclassical model.

   (b) Given that π^e_{t+1} is taken to be exogenous, show that the nominal interest rate peg translates into a constant target value of the real interest rate.

   (c) Now graphically analyze the model. Suppose that the economy initially begins in an equilibrium and that there is then an exogenous increase in f_t (i.e. a worsening of financial conditions). How must the money supply change in order to maintain the interest rate peg? Show how the equilibrium changes. Do the endogenous variables change more or less in comparison to the standard version of the model in which there is no interest rate peg and the money supply is instead exogenous?

   (d) Suppose that the central bank's objective is to implement the neoclassical equilibrium – i.e. to use monetary policy to implement $Y_t = Y_t^f$ as the equilibrium outcome of the sticky price model. If so, based on your previous answers do you think an interest rate peg is a good idea? Explain.

5. **A Flexible Price Level Target:** Consider the standard sticky price New Keynesian model as presented in class. The key equations of the model are:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$
$$P_t = \bar{P}_t + \gamma(Y_t - Y_t^f)$$
$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$
$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t + G_t$$

$$M_t = P_t M^d(i_t, Y_t)$$

$$r_t = i_t - \pi^e_{t+1}$$

In the baseline model, we assume that M_t is exogenous. Let us instead assume that M_t is instead endogenous. In particular, suppose that the central bank targets a price level of P_t^*. Hence, we add one equation to the above system of equations:

$$P_t = P_t^*$$

P_t^* is exogenous, and now M_t is endogenous.

(a) Argue that if $P_t^* = \bar{P}_t$, then the equilibrium will feature $Y_t = Y_t^f$.

(b) Suppose that this is the rule the central bank follows. Suppose that there is an exogenous increase in A_t. In which direction must M_t adjust to implement the price level target? Explain briefly.

(c) Instead suppose that there is an exogenous increase in G_t. In which direction must M_t adjust to implement the price level target?

(d) Suppose that the central bank follows a price level target, but the target is exogenous and does not necessarily correspond to $\bar{P}_t$. Suppose that $\bar{P}_t$ changes but P_t^* does not. What will happen to Y_t and M_t as a consequence? Explain briefly.