

# Money Demand

ECON 30020: Intermediate Macroeconomics

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# Readings

GLS Ch. 14

# What is Money?

Might seem like an obvious question but really not so clear

Money is an asset – it's a stock that can be used to transfer resources across time

Money a special kind of asset in that it can be used in exchange

# Functions of Money

With most things we define them according to intrinsic characteristics (e.g., coffee is a dark, liquid substance)

With money we instead give a functional definition

Money is any asset that serves the following three functions:

1. Medium of exchange
2. Store of value
3. Unit of account

## Medium of Exchange

The most important role played by money is its role as a medium of exchange

This solves the “double coincidence of wants” problem associated with barter

Bonds and capital can serve as stores of values (any asset does so, so money is not unique), and anything can serve as a unit of account in principle

But money is unique in its role as medium of exchange

Money's role as a medium of exchange has been critical to the historical growth in economic activity via specialization

Fiat money is the best medium of exchange, so long as people believe it has value

# Including Money in the Neoclassical Model

Is not so easy

Why? Model only features one good (e.g. fruit). Makes medium of exchange role uninteresting

We will include money essentially as an asset (i.e., a store of value) and will use money as a nominal unit of account

But money is a crummy store of value – bonds pay interest, money does not

We will therefore take a reduced-form shortcut and assume that the household receives utility from holding money

## New Nominal Variables

Real variables are denominated in units of goods (i.e., fruit)

Nominal variables are denominated in units of money (i.e., dollars)

New variables:

- ▶  $M_t$ : stock of money held between periods  $t$  and  $t + 1$  (i.e. store of value like  $S_t$ )
- ▶  $P_t$ : price of goods measured in units of money
- ▶  $i_t$ : nominal interest rate (as a rate, this is actually unitless)

## Nominal Budget Constraints

$$P_t C_t + P_t S_t + M_t \leq P_t w_t N_t - P_t T_t + P_t D_t$$

$$P_{t+1} C_{t+1} + P_{t+1} S_{t+1} - P_t S_t + M_{t+1} - M_t \leq \\ P_{t+1} w_{t+1} N_{t+1} - P_{t+1} T_{t+1} + i_t P_t S_t + P_{t+1} D_{t+1} + P_{t+1} D'_{t+1}$$

Terminal conditions:  $S_{t+1} = 0$  and  $M_{t+1} = 0$ . Writing constraints in real terms:

$$C_t + S_t + \frac{M_t}{P_t} = w_t N_t - T_t + D_t$$

$$C_{t+1} = w_{t+1} N_{t+1} - T_{t+1} + D_{t+1} + (1 + i_t) \frac{P_t}{P_{t+1}} S_t + D'_{t+1} + \frac{M_t}{P_{t+1}}$$

$\frac{M_t}{P_t}$ : real money balances



## Fisher Relationship

The Fisher relationship connects the real and nominal interest rates:

$$1 + r_t = (1 + i_t) \frac{P_t}{P_{t+1}}$$

Expected inflation between  $t$  and  $t + 1$ :

$$1 + \pi_{t+1}^e = \frac{P_{t+1}}{P_t}$$

Fisher relationship is then approximately:

$$r_t = i_t - \pi_{t+1}^e$$

We will treat expected one period ahead inflation rate,  $\pi_{t+1}^e$ , as exogenous. Movements and nominal and real rates are the same for a given rate of expected inflation

# The Real Intertemporal Budget Constraint

Can write  $t + 1$  constraint as:

$$C_{t+1} = w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + D'_{t+1} + (1 + r_t)S_t + \frac{1 + r_t}{1 + i_t} \frac{M_t}{P_t}$$

Solve out for  $S_t$ , combining with period  $t$  constraint:

$$C_t + \frac{C_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t} = w_t N_t - T_t + D_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + D'_{t+1}}{1 + r_t}$$

Exactly the same as before, just this additional “expenditure” category of  $\frac{M_t}{P_t}$  – how many period  $t$  goods held in the form of money

## Preferences

Note that money is held across periods, not within a period (i.e. it is a stock variable, not a flow)

Assume household receives a utility flow from its holding of real balances via the function  $v(\cdot)$ . Increasing and concave (e.g., log)

This utility flow is received in period  $t$ . Lifetime utility:

$$U = u(C_t, 1 - N_t) + v\left(\frac{M_t}{P_t}\right) + \beta u(C_{t+1}, 1 - N_{t+1})$$

## Optimality Conditions

FOC for consumption and labor exactly the same as before:

$$u_C(C_t, 1 - N_t) = \beta(1 + r_t)u_C(C_{t+1}, 1 - N_t)$$

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$

New FOC for money:

$$v' \left( \frac{M_t}{P_t} \right) = \frac{i_t}{1 + i_t} u_C(C_t, 1 - N_t)$$

Interpretation ... marginal benefit equals marginal cost!

If no utility benefit from holding money ( $v'(\cdot) = 0$ ), then could only hold if  $i_t = 0$ : money dominated as a store of value by bonds if  $i_t > 0$

## Optimal Decision Rules

Presence of money does not impact optimal decision rules for consumption or labor supply:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$
$$N_t = N^s(w_t, \theta_t)$$

Cutting a few corners (i.e. treating  $C_t$  and  $Y_t$  as interchangeable), optimal decision rule for money is:

$$M_t = P_t M^d(\underset{-}{i_t}, \underset{+}{Y_t})$$

# Money Demand Function

Equivalently, using the Fisher relationship:

$$M_t = P_t M^d(r_t + \underset{-}{\pi_{t+1}^e}, \underset{+}{Y_t})$$

This is our money demand function – demand for real balances is decreasing in the nominal rate and increasing in total expenditure

## Government

Government “prints” money, and we take this to be exogenous.  
Period  $t$  budget constraint:

$$P_t G_t \leq P_t T_t + P_t B_t + M_t$$

Government can use money as an additional “revenue” source (way to finance spending). Period  $t + 1$  constraint:

$$P_{t+1} G_{t+1} + i_t P_t B_t + M_t \leq P_{t+1} T_{t+1} + P_{t+1} B_{t+1} - P_t B_t$$

Government essentially has to “buy back” in period  $t + 1$  the money it issues in period  $t$ . Terminal condition:  $B_{t+1} = 0$ , implying:

$$P_{t+1} G_{t+1} + (1 + i_t) P_t B_t + M_t \leq P_{t+1} T_{t+1}$$

## Government's IBC

In real terms, the two flow budget constraints for the government are:

$$G_t = T_t + B_t + \frac{M_t}{P_t}$$
$$G_{t+1} + (1 + i_t) \frac{P_t}{P_{t+1}} B_t + \frac{M_t}{P_{t+1}} = T_{t+1}$$

Combining the two and using the Fisher relationship, we get:

$$G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t} \frac{M_t}{P_t}$$

Similar to before, but additional “revenue” category related to money (what we call seignorage). Analogous to household IBC which features the same term but as an expenditure category

When combining firm and household IBCs, these terms cancel



# Equilibrium Conditions

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$

$$N_t = N^s(w_t, \theta_t)$$

$$N_t = N^d(w_t, A_t, K_t)$$

$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t + G_t$$

$$M_t = P_t M^d(i_t, Y_t)$$

$$r_t = i_t - \pi_{t+1}^e$$

# Classical Dichotomy

Eight endogenous variables:  $Y_t$ ,  $C_t$ ,  $I_t$ ,  $N_t$ ,  $w_t$ ,  $r_t$ ,  $P_t$ , and  $i_t$

New exogenous variables:  $M_t$  and  $\pi_{t+1}^e$

First six equations feature six real endogenous variables and no nominal variables

- ▶ Means that the real endogenous variables are determined independently of nominal variables
- ▶ This is known as the *classical dichotomy*
- ▶ Do not need to know nominal variables to determine real variables
- ▶ Converse not true: nominal variables will be affected by real variables

## Graphing the Equilibrium

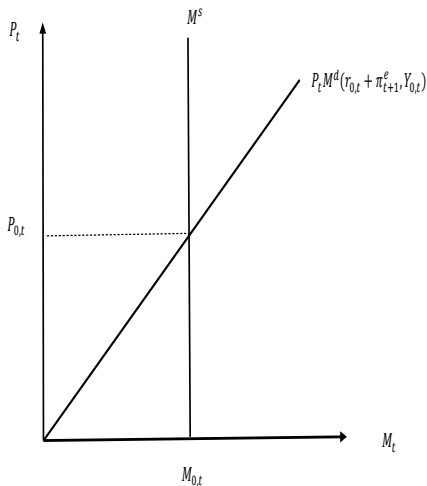
Can use the same five-part graph as before to determine equilibrium of the real side of the economy

The real interest rate,  $r_t$ , and output,  $Y_t$ , are relevant for money demand

Once we know  $r_t$  and  $Y_t$ , along with the exogenous quantity of money supplied, can determine  $P_t$

Given an exogenous  $\pi_{t+1}^e$ , given  $r_t$  can determine  $i_t$  ( $i_t$  and  $r_t$  always move in same direction absent a change in  $\pi_{t+1}^e$ )

# Money Market Equilibrium



Looks funny to have “demand” upward-sloping, but  $P_t$  is price of goods in terms of money, so  $\frac{1}{P_t}$  is price of money in terms of goods. Demand decreasing in  $\frac{1}{P_t}$

## Monetary Neutrality

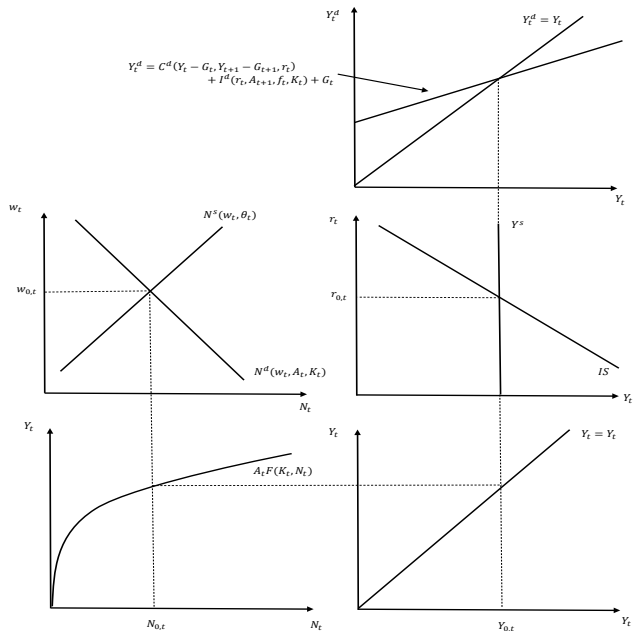
Increase in  $M_t$  does not affect first six equations – no effect of change in  $M_t$  on any real endogenous variable

We say that money is neutral

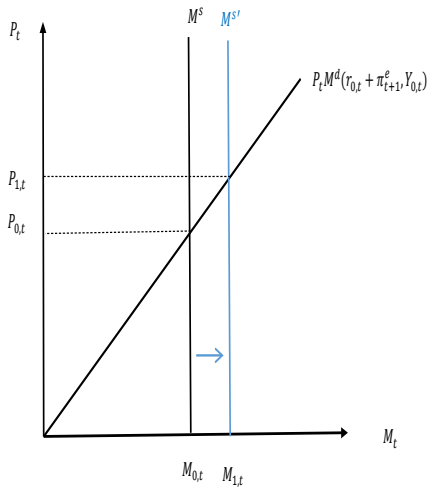
Useful medium-run benchmark, but in the short run nominal rigidities may break monetary neutrality

In this model, only effect of an increase in  $M_t$  is an increase in  $P_t$

# Increase in $M_t$



# Increase in $M_t$



## Real Shocks Affect Nominal Variables

Increase in  $A_t$ : lowers  $r_t$  and raises  $Y_t$ , both of which pivot money demand to the right, and hence lower  $P_t$

Increase in  $\theta_t$ : raises  $r_t$  and lowers  $Y_t$ , both of which pivot money demand to the left, and hence raise  $P_t$

Positive “demand” shocks (increases in  $A_{t+1}$  or  $G_t$ , or decreases in  $f_t$  or  $G_{t+1}$ ): raise  $r_t$ , no effect on  $Y_t$ . Hence, money demand shifts left, and price level rises

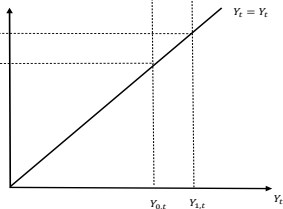
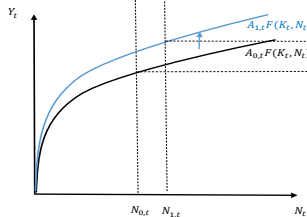
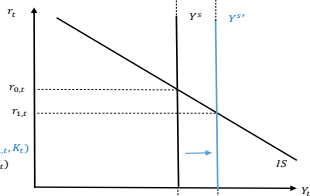
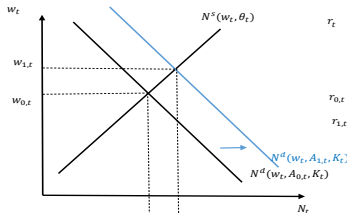
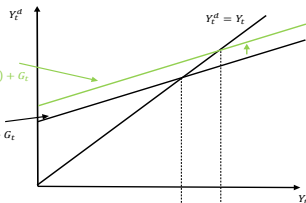
Increase in  $\pi_{t+1}^e$ :  $i_t$  rises by same amount. Money demand pivots in, so price level increases. “Self-fulfilling” inflation



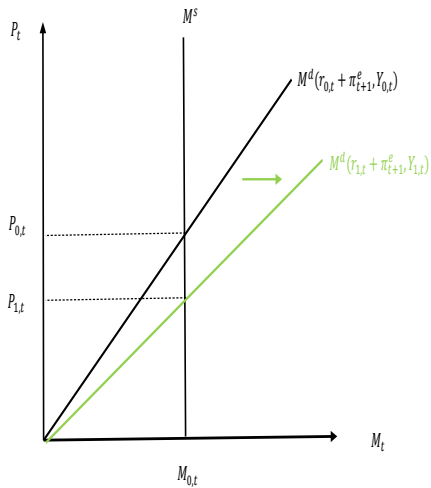
# Supply Shock: $\uparrow A_t$

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{1,t}) + I^d(r_{1,t}, A_{t+1}, f_t, K_t) + G_t$$

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t}) + I^d(r_{0,t}, A_{t+1}, f_t, K_t) + G_t$$



# Increase in $A_t$

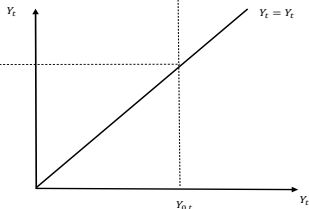
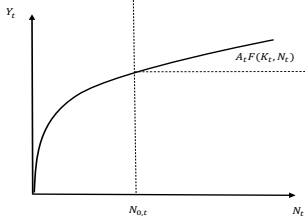
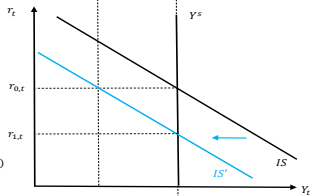
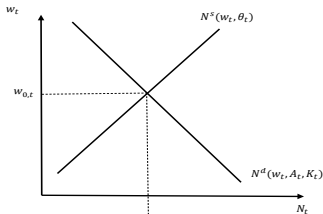
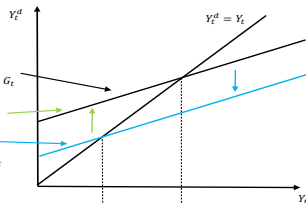


# Demand Shock: $\uparrow f_t$

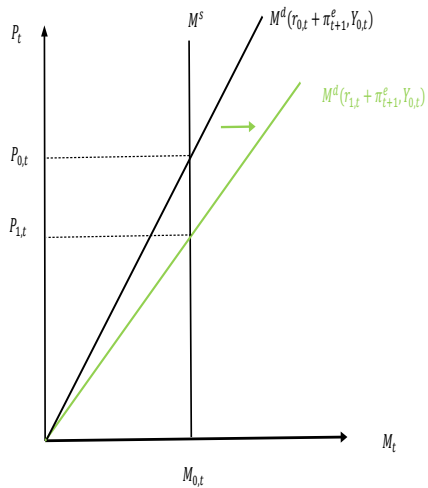
$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t}) + I^d(r_{0,t}, A_{t+1}, \bar{f}_{0,t}, K_t) + G_t$$

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{1,t}) + I^d(r_{1,t}, A_{t+1}, \bar{f}_{1,t}, K_t) + G_t$$

$$Y_t^d = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_{0,t}) + I^d(r_{0,t}, A_{t+1}, \bar{f}_{1,t}, K_t) + G_t$$



# Increase in $f_t$



# Qualitative Effects

Variable	Exogenous Shock							
	$\uparrow A_t$	$\uparrow \theta_t$	$\uparrow f_t$	$\uparrow A_{t+1}$	$\uparrow G_t$	$\uparrow G_{t+1}$	$\uparrow M_t$	$\uparrow \pi_{t+1}^e$
$Y_t$	+	-	0	0	0	0	0	0
$C_t$	+	-	+	?	-	-	0	0
$I_t$	+	-	-	?	-	+	0	0
$N_t$	+	-	0	0	0	0	0	0
$w_t$	+	+	0	0	0	0	0	0
$r_t$	-	+	-	+	+	-	0	0
$i_t$	-	+	-	+	+	-	0	+
$P_t$	-	+	-	+	+	-	+	+