Problem Set 3

ECON 40364: Monetary Theory and Policy
Prof. Sims
Spring 2025

Instructions: Please answer all questions to the best of your ability. You may consult with other members of the class, but each student is expected to turn in his or her own assignment. This problem set is due in class on Wednesday, March 5, 2025.

1. In the AD-AS model from class we abstract from money altogether. However, it is possible to think about how the central bank must adjust the money supply, given its monetary policy rule expressed in terms of the real interest rate. The main equations of the model (abstracting from money) assuming adaptive expectations are:

$$\text{(IS)} \quad Y_t = \frac{1}{1 - mpc}\left[\bar{C} - mpc\bar{T} + \bar{I} + \bar{G} + \bar{NX}\right] - \frac{d + x}{1 - mpc}r_t$$

$$\text{(MP)} \quad r_t = \bar{r} + \lambda\pi_t$$

$$\text{(AS)} \quad \pi_t = \pi_{t-1} + \gamma(Y_t - Y^P) + \rho$$

Here we are assuming that exogenous variables Y^P, ρ, $\bar{r}$, and the components of $\bar{A}$ are constant (unless you are otherwise told they change, in which case they would change permanently).

(a) Combine these three equations to solve for algebraic expressions for the three endogenous variables (Y_t, π_t, r_t) in terms of exogenous variables $(\pi_{t-1}, Y^P, \rho, \bar{r}$, and the autonomous components of expenditure, which you may simplify to just $\bar{A}$).

(b) Create an Excel file in which to conduct simulations. Assume the following parameter values:

$$mpc = 0.7$$
$$d = 0.3$$
$$x = 0.1$$
$$\lambda = 1$$
$$\gamma = 0.5$$

Assume that the exogenous variables are *constant* (and hence no time subscripts) and take on the following values:

$$\bar{r} = 1$$
$$\bar{C} = 3.25$$
$$\bar{I} = 1.3$$
$$\bar{G} = 3.5$$
$$\bar{T} = 3$$

$$\bar{NX} = -1$$
$$\bar{\rho} = 0$$
$$Y^P = 12.5$$

Compute a simulation running from periods −5 through 10. In period −5, assume that lagged inflation (i.e. π_{-6}) is 2. Starting in period −4, you will use the value of inflation you found in the previous period to solve for inflation in the present period. Plot the simulated values of Y, π, and r from periods −5 to 10.

(c) Now do another simulation. In this simulation, suppose that, in period 0, businesses are imbued with "animal spirits," and $\bar{I}$ increases from 1.3 to 1.6. It is expected to remain forever higher at 1.6. Simulate Y, π, and r from periods −5 to 10 with this change in $\bar{I}$ occurring in period 0 (and remaining in effect for all subsequent periods). Please plot the *differences* between the simulated values of Y, π, and r from the simulation with the change in $\bar{I}$ relative to the simulation without the change in $\bar{I}$.

(d) Now, let's think about what happens to the money supply here. Suppose that money demand is given by the Baumol-Tobin model. In particular, assume that the demand for *real balances*, $m_t = \frac{M_t}{P_t}$, is given by:

$$m_t = \left(\frac{KY_t}{2}\right)^{\frac{1}{2}} (r_t + \pi_t)^{-\frac{1}{2}}$$

Here, $r_t + \pi_t$ is the nominal interest rate, i_t, assuming adaptive expectations. K is a parameter. Assume that its value is 2.

 i. Create a column in your Excel sheet to solve for simulated values of m_t in both simulations (the one without the change in $\bar{I}$ as well as the one with the change in $\bar{I}$).

 ii. Suppose that the price level in period −6 is 1. This means that the price level in period −5 is $P_{-5} = (1 + 0.01 \times \pi_{-5}) \times 1$, where the multiplication by 0.01 is necessary because we are working with growth rates. Then the price level in period −4 would be $(1 + 0.01 \times \pi_{-4}) \times P_{-5}$, and so on for the remainder of the periods. Create a column for the simulated price level in both of your simulations.

 iii. Given simulated values of m and P, you can recover the simulated value of the money supply, M, as the product of the two. Create a column for the simulated money supply in both of your simulations.

Please plot the difference between the simulated values of the money supply from the simulation with the change in $\bar{I}$ relative to the simulation without the change in $\bar{I}$. How does the money supply react to the increase in $\bar{I}$? Does it go up or down? What is your intuition for this?

(e) Re-do parts (c)-(d). But for this part, assume that $\bar{I}$ does not change in period 0 (it remains at 1.3). Rather, assume that Y^P increases from 12.5 to 13 in period 0 (and is expected to remain forever there).

(f) Compare and contrast your findings for the path of the money supply conditional on the two shocks (the "demand" shock when $\bar{I}$ increases and the "supply" shock when Y^P increases). Some people argue that central banks should reduce the money supply any time output increases. Is this consistent with what you find?

2. **Fun with Bond Pricing:** Please provide answers (with justification) to the following bond pricing problems. Since this problem requires explicit calculations, you may find it advantages to use a program like Excel or Matlab to do the calculations. You may also find it advantageous for some parts to use Excel's equation solver (Google "Excel equation solver" to see how to add it in and use it). Alternatively, you can just use a conventional calculator.

   (a) Suppose a bond has a face value of 1000, a coupon rate of 6 percent, and matures in 5 years. If the yield to maturity is 8 percent, what is the bond's current price?

   (b) Suppose a bond has a face value of 1000, a coupon rate of 10 percent, and matures in 4 years. Its current price is 1140. What is the bond's yield to maturity?

   (c) Suppose a bond has a current price of 800, a face value of 1000, and matures in 5 years. The yield to maturity is 8 percent. What is the bond's coupon rate?

   (d) Suppose there are two discount bonds, A and B. Both have face values of 1000. Bond A has a time to maturity of 1 year, and Bond B has a time to maturity of 5 years. The yield to maturity on both bonds is initially 5 percent. Calculate the prices of both bonds.

   (e) Continue with the previous problem. Suppose that the yield to maturity were instead 10 percent. Calculate the prices of both bonds A and B. Which bond's bond price is affected more by the change in yield to maturity?

   (f) Suppose that you have a bond with a face value of 1000, a coupon rate of 5 percent, 10 years to maturity, and a yield to maturity of 7 percent. Calculate the bond's price as well as its current yield. How does the current yield compare to the yield to maturity?

   (g) Re-do the previous part, but this time with a time to maturity of 30 years. Comment on whether the current yield is closer to the yield to maturity in this part relative to the previous part.

   (h) Suppose that you have a 1000 face value discount bond that has a yield of 10 percent and has 10 years remaining to maturity in period t. Calculate its price. Now suppose that a year from now, $t+1$, the yield decreases to 5 percent. Calculate the new price. Calculate the realized return on the bond going from t to $t+1$.

   (i) Re-do the previous part, but assume that the bond initially has 20 years remaining to maturity instead of 10. Does the realized return from the drop in yield in period $t+1$ move more or less with a longer time to maturity?

3