Problem Set 4

ECON 40364: Monetary Theory and Policy
Prof. Sims
Spring 2025

Instructions: Please answer all questions to the best of your ability. You may consult with other members of the class, but each student is expected to turn in his or her own assignment. This problem set is due in class on Monday, March 31, 2025.

1. **Determinants of Bond Prices:** Please use bond demand-supply diagrams to predict the likely effects of the following changes on bond yields and prices. Please provide a brief, written explanation for each part.

   (a) The public expects a large increase in the valuation of stock market in the near future.

   (b) A tax policy change makes it less profitable for firms to issue bonds.

   (c) The government runs a massive budget deficit.

   (d) People expect future short-term interest rates to increase.

2. **The Bond Risk Premium** Consider a model with a representative household. The household can save (or borrow) with one-period discount bonds, $B_{1,t}$ and $B_{2,t}$. The face value of both bonds is 1, and they trade at prices $P_{1,t}$ and $P_{2,t}$. $B_{1,t}$ is risk-free: purchasing $B_{1,t}$ units of this bond in period t guarantees $B_{1,t}$ units of income in the future (period $t+1$). $B_{2,t}$ is risky – in some states of the world, it pays face value in $t+1$; in other states of the world, it defaults (completely, so that it pays nothing).

   The household is endowed with current income $Y_t = 1$. this is known. Future income is unknown. In some states of the world, it is high; in others, it is low.

   Suppose that there are exactly four states of the world, each with probabilities of occurring p_1, p_2, p_3, and p_4, where $p_1 + p_2 + p_3 + p_4 = 1$. These four states are summarized below:

$$\begin{aligned} 1: &\ Y_{t+1} = 1.1, \text{ bond 2 defaults} \\ 2: &\ Y_{t+1} = 1.1, \text{ bond 2 pays} \\ 3: &\ Y_{t+1} = 0.9, \text{ bond 2 defaults} \\ 4: &\ Y_{t+1} = 0.9, \text{ bond 2 pays} \end{aligned}$$

   In the first period, the household faces the flow budget constraint:

$$C_t + P_{1,t}B_{1,t} + P_{2,t}B_{2,t} = 1$$

   In the second period, a flow budget constraint must hold *in each state of the world.* That is, we must have:

$$C_{t+1}(1) = 1.1 + B_{1,t}$$

$$C_{t+1}(2) = 1.1 + B_{1,t} + B_{2,t}$$

$$C_{t+1}(3) = 0.9 + B_{1,t}$$

$$C_{t+1}(4) = 0.9 + B_{1,t} + B_{2,t}$$

$C_{t+1}(j)$, $j = 1, \dots 4$, denotes consumption in each possible state of the world in the future. The household maximizes expected discounted lifetime utility:

$$U = \ln C_t + \beta \mathbb{E} \ln C_{t+1} = \ln C_t + \beta \left[p_1 \ln C_{t+1}(1) + p_2 \ln C_{t+1}(2) + p_3 \ln C_{t+1}(3) + p_4 \ln C_{t+1}(4) \right]$$

(a) Use calculus to derive the optimal first-order conditions for this problem.

(b) What is the stochastic discount factor? Explain, in words, why it is stochastic and why it is used to "price" assets.

(c) Suppose that $\beta = 0.9$. Suppose further that $p_1 = p_2 = p_3 = p_4 = 1/4$. Suppose that the supply of bond 1 is fixed at $B_{1,t} = 0.1$ and the supply of bond 2 is fixed, also at $B_{1,t} = 0.1$. Solve for the prices of both bonds.

(d) Express your answers from part (c) in terms of yield to maturity. What is the spread between the yield on the risky bond and the yield on the riskless bond. Can you provide some intuition for your answer?

(e) Re-do parts (c) and (d), but this time assume that both bonds are in zero supply, that is $B_{1,t} = B_{2,t} = 0$. Provide some intuition for how the prices of both bonds change relative to part (b). What is the yield spread now? Please provide some intuition for if and why your answer differs from (c).

(f) Go back to the setup in parts (c) - (d) (i.e., the supply of both bonds is fixed at $B_{1,t} = B_{2,t} = 0.1$). But instead assume that the probabilities of the states are different. In particular, suppose that $p_2 = 0.5$ and $p_3 = 0.5$, with $p_1 = p_4 = 0$. First, verify that the expected value of future income is the same as in part (c), as is the expected value of the payout on the bond. Then, re-compute the bond prices, yields, and yield spread. Provide a brief rational for how your answers differ relative to parts (c) and (d).

(g) Continue with the original set up (i.e., the supply of both bonds is fixed at $B_{1,t} = B_{2,t} = 0.1$). But now assume that $p_1 = p_4 = 0.5$, while $p_2 = p_3 = 0$. Again, verify that the expected values of future income and the payout are the bond are the same as in the earlier part of the problem. Then, re-compute the bond prices, yield, and yield spread. Provide a brief rationale for how your answers differ relative to parts (c) and (d) (and part (f)).

3. **Yield Curves:** Please go to the following website to access historical daily yields on US government debt of different maturities. You should be collecting data on "Daily Treasury Par Yield Curve Rates." Please use Microsoft Excel for all calculations which follow.

   (a) Create a table showing the yields on Treasury securities with maturities of 1, 2, 3, 5, 7, 10, and 20 year maturities on the following specific dates:

      i. December 31, 1993
      ii. April 20, 1995
      iii. December 27, 2000
      iv. April 23, 2004

v. May 7, 2007

vi. January 3, 2014

vii. March 3, 2020

(b) Create a plot of the yield curve for each of these dates. What does the yield curve "normally" look like? What are a couple of dates where the yield curve looks "different"? What was happening around those "different dates"? What about March 2020?

(c) For each of the given dates, use data on the 1, 2, and 3 year maturity yields to infer market expectations of one year interest rates 1 and 2 years ahead (i.e. back out the one year "forward rates" for each date). In which years was the market expecting rates to decline versus increase? Is your answer consistent with your plots of the yield curves?