Problem Set 5

ECON 40364: Monetary Theory and Policy
Prof. Sims
Spring 2025

Instructions: Please answer all questions to the best of your ability. You may consult with other members of the class, but each student is expected to turn in his or her own assignment. This problem set is due in class on Monday, April 28, 2025.

1. **The Gordon Growth Model:** Suppose that there is a stock which currently pays a dividend of 1 ($D_t = 1$). It is expected that dividends will grow into the future at a constant rate of $g = 0.02$ and will do so forever (i.e. $D_{t+h} = (1+g)^h D_t$ for $h \geq 0$. The discount rate for equity is constant $\kappa^e = 0.05$.

   (a) Imposing a no-bubble condition, solve for the price of the stock in period t.

   (b) What is the expected price of the stock in $t+1$? What is the expected return? Decompose the expected return into dividend and capital gain.

   (c) For the general case (i.e. use symbols, not actual numbers), derive an expression for the dividend component of the expected return (as a function of κ^e and g) and the capital gain component of the expected return (as a function of κ^e and g). As g gets bigger, which term – dividends or capital gains – drives a bigger component of the total expected return?

2. **The Equity Risk Premium:** The generic pricing formula for a share of stock from a micro-founded model is as follows:

$$P_t = \mathbb{E}\left[m_{t,t+1}\left(D_{t+1} + P_{t+1}\right)\right]$$

Here, $m_{t,t+1} = \frac{\beta u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor, D_{t+1} is the future dividend, and P_{t+1} is the future share price. P_t is the current share price. Define the expected return on equity (or the discount rate) as:

$$P_t = \mathbb{E}\left(\frac{D_{t+1} + P_{t+1}}{1 + \kappa_e}\right)$$

Suppose that the period flow utility function is the natural log:

$$u(C_t) = \ln C_t$$

   (a) With this utility function, show that one can write the stochastic discount factor as a function of the gross growth rate of consumption between periods t and $t+1$, i.e., $1 + g_{t+1} = \frac{C_{t+1}}{C_t}$.

   (b) Suppose that $C_t = 1$ and $\beta = 0.95$. C_{t+1} can take on two values, 1.2 and 0.8, each with probability 1/2. What is the expected value of future consumption, $\mathbb{E}C_{t+1}$? What is the expected value of the stochastic discount factor, $\mathbb{E}m_{t,t+1}$?

(c) Suppose that the expected value of future cash flows is $\mathbb{E}(D_{t+1} + P_{t+1}) = 10$. Suppose further than the covariance between future cash flows and the stochastic discount factor, i.e., $\text{cov}\,(m_{t,t+1}, D_{t+1} + P_{t+1}) = 0$. Calculate the price of the stock, P_t, and the expected return on the stock, κ_e.

(d) Suppose instead that the covriance between future cash flows and the stochastic discount factor is $\text{cov}\,(m_{t,t+1}, D_{t+1} + P_{t+1}) = -0.5$. Calculate the price of the stock, P_t, and the expected return on the stock, κ_e.

(e) Explain, in words only, why the expected return on the stock is different in part (d) than in part (c)? In particular, what explains the sign of the difference?

3. **Bubbles:** Suppose that you have a stock that currently pays a dividend of $D_t = 1$. Future dividends are discounted at a constant rate of $\kappa^e = 0.07$, and dividends are expected to grow at a constant rate forever of $g = 0.02$. The current price of the stock is $P_t = 25$.

   (a) What is the magnitude of the bubble term for this stock?

   (b) Given your answer on (a), what would you expect the price of the stock to be in $t + 1$?

   (c) Given your answers on (a)-(b), what is your expected return from holding this stock from t to $t + 1$?

   (d) Suppose that the bubble bursts in period $t + 1$. What is your realized return (not your expected return) from holding this stock from t to $t + 1$?

   (e) Suppose that the bubble bursts with probability 1/10. Suppose that the bubble does not burst in period $t + 1$. What is your realized return (not your expected return) from holding this stock from t to $t + 1$?

4. **Adverse Selection and Collateral:** Suppose that there is one bank and two types firms that seek funding of 1 for a project. The two types of firms are safe and risky. If a safe firm undertakes a project, it will succeed and earn 2 with probability 4/5; with probability 1/5 it will earn nothing. If a risky firm undertakes a project, it will earn 3 if the project succeeds, but the project only succeeds with probability 1/6; with probability 5/6 the project fails and the firm earns nothing.

   Assume that there is a bank that has funds. It can lend these funds to either kind of firm at a (gross) interest rate of R. The bank's opportunity cost of funds is 1 – it will not make a loan unless it expects to get back at least 1. There is limited liability – if a project fails a firm owes the bank nothing, whereas if the project succeeds a firm owes the bank R.

   (a) Suppose that the bank could tell safe firms apart from risky firms and charge them different interest rates. Suppose that the market structure is such that if the bank makes a loan, it just breaks even (i.e. earns 1 in expectation, its opportunity cost). Will both types of firms be able to secure funding? If not, which type of firm will be able to secure funding?

   (b) Now suppose that the bank cannot tell safe firms apart from risky firms. It only knows that 50 percent of firms are safe, and 50 percent are risky. It can only post one loan contract. Please plot the lender's expected payout (vertical axis) against R. What will happen in this market? Will both firms be able to secure funding? If not, which one type of firm be able to secure funding? Explain briefly.

(c) Now suppose that the lender can require a firm to post collateral in the amount of $C = \frac{1}{2}$ on a loan contract – should a project be undertaken and not succeed, the lender can get back the collateral. If the bank must just break even, will either firm (or both) be able to secure funding? If so, which one? Justify your answer.

5. **Banking**: Suppose that First Source Bank has the following balance sheet:

Assets	Liabilities plus Equity
Loans: \$100	Deposits: \$90
Securities: \$10	Borrowings: \$20
Cash Reserves: \$10	Equity: \$10

Loans are illiquid. If they are held, each period they earn 20 percent interest. But if First Source has to sell a lona, they must do so at a 50 percent discount (i.e. selling 1 dollar of loans generates only 0.50 in cash). Securities earn 10 percent interest if held and can be sold dollar-for-dollar in the event the bank needs to raise cash. The bank pays no interest on deposits. If pays 10 percent interest on borrowings.

(a) What is the bank's leverage ratio?

(b) If the bank sells no loans or securities, has no withdrawals or extra deposits, and does no additional borrowing, what will be its profit? What will be its return on equity?

(c) Suppose that the bank's initial balance sheet is as shown above. Suppose that it faces a 20 dollar withdrawal. Assuming it cannot adjust its borrowings, how will the bank adjust the asset side of its balance sheet to meet the withdrawal? How will the bank's equity be affect? Show in a new T-Account.

(d) Now assume the initial balance sheet is what you end up finding in part (c), but that there is another 20 dollar withdrawal. Assuming it cannot adjust its borrowings, how will the bank adjust the asset side of its balance sheet to meet the withdrawal demand? How will the bank's equity be affected? Show in a new T-Account.

(e) Based on your answers above, discuss the pros and cons of holding relatively more liquidity on the asset side of the balance sheet (i.e. more securities and cash compared to loans).

6. **Bank Runs**: Suppose that there are 1000 households with 1 dollar each. These households live for three periods: 0, 1, and 2. The households have no need for consumption in period 0. A fraction $\frac{1}{2}$ of households will need to consume in period 1; the other fraction $\frac{1}{2}$ have the option of waiting to consume until period 2. In period 0, a household does not know whether it will need to consume in period 1 or can wait until period 2.

There is an illiquid long term investment opportunity which costs 1 dollar to invest in. If held until period 2, the project offers a gross return of 1.8. If liquidated in period 1, the project only returns $\frac{1}{2}$. Suppose that the household's utility function if $\ln C$. If a household chooses to not to invest its 1 dollar, it can simply store it and have consumption $C = 1$ in whatever period (1 or 2) the household needs to consume.

(a) What is the expected (gross) return on the investment opportunity? Is this higher or lower than the expected (gross) return on storage?

(b) Suppose that each household gets utility from consumption given by the function $U = \ln C$. Calculate the household's utility from storing its 1 dollar as well as the household's expected utility from investing in the project. Will the long run project get funded directly by households?

(c) Now suppose that there is a mutual bank which takes deposits from the 1000 households. The bank anticipates that 500 households will need to withdraw their money in period 1. The bank promises households who withdraw in period 1 a gross return of $r_1 = 1.1$. How much of the 1000 dollars in deposits it receives should the bank store in its vault, and how much should it invest in the investment project? What return can the bank promise households who withdraw their money in period 2 (i.e. what is r_2)?

(d) Calculate the household's expected utility from depositing with the bank and verify that it is greater than expected utility from storage.

(e) What is the maximum number of additional withdrawals (additional meaning withdrawals from depositors who do not have to consume) in period 1 before the bank will be unable to pay back deposits the promised r_1?

(f) Suppose that you are a household who does not have to consume in period 1. If you expect $N \geq 0$ other "patient" households to withdraw their money in period 1, it would also make sense for you to withdraw. Solve for N.