Bonds, Bond Prices, Interest Rates, and the Risk and Term Structure of Interest Rates

ECON 40364: Monetary Theory & Policy

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Readings

Mishkin Ch. 4; Mishkin Ch. 5, Sections 5.1, 5.2, and 5.3; Mishkin Ch. 6

GLS Ch. 34

Poole (2005): "Understanding the Term Structure of Interest Rates"

Bernanke (2016), "What Tools Does the Fed Have Left? Part 2: Targeting Longer-Term Interest Rates"

Bonds

Generically, a financial security entitles the holder to periodic cash flows and obligates issuer to make those cash flows (i.e., a security is both an asset and a liability)

We will generically refer to a "bond" as a debt instrument where a borrower promises to pay the holder of the bond (the lender) fixed and known payments according to some pre-specified schedule

Hence, bonds are known as fixed-income securities

In contrast, equity (stocks) offer unknown payout streams

Types of Bonds and YTM

There are many different types of bonds. Differ according to:

- Details of how bond is paid off
- Time to maturity
- Default risk

The yield to maturity (just "yield" for short) is a measure of the implied interest rate on the bond, although the interest rate is often not explicitly laid out

Will use terms interest rate and yield interchangeably

Want to understand how interest rates are determined and how and why they vary across different characteristics of bonds

Present Value

Present discounted value (PDV): something received in the future is worth less to you than if you received the thing in the present

Discount rate (sometimes required return): rate at which you discount future cash flows from a security. Suppose the discount rate is i and is constant

For a future cash flow (CF), how many dollars would be equivalent to you today:

$$PV_t = \frac{CF_{t+n}}{(1+i)(1+i)(1+i)\cdots \times (1+i)}$$

Here t is the "present," $t+n$ is the future (n periods away), and i is the discount rate

$$PV_t = \frac{CF_{t+n}}{(1+i)^n}$$

Present Value: Example I

Suppose you are promised \$10 in period $t + 1$

Outside option: you could put \$1 in bank in period t and earn $i = 0.05$ in interest between t and $t + 1$

How many dollars would you need in the present to have \$10 in the future?

$$
\begin{aligned}
(1 + i)PV_t &= CF_{t+1} \\
PV_t &= \frac{CF_{t+1}}{1 + i} \\
&= \frac{10}{1.05} = 9.5238
\end{aligned}
$$

Present Value: Example II

Suppose you are promised \$10 in period $t+3$

Outside option: you could put \$1 in bank in period t and earn $i = 0.05$ across each set of adjacent periods between now and then

If you put \$1 in bank in period t and kept it there (re-investing any interest income), you would have $(1+i)(1+i)(1+i)$ dollars in $t+3$. Hence, the present value of \$10 three periods from now is:

$$
\begin{aligned}
PV_t(1+i)(1+i)(1+i) &= CF_{t+3} \\
PV_t &= \frac{CF_{t+3}}{(1+i)^3} \\
&= \frac{10}{1.05^3} = 8.6384
\end{aligned}
$$

Present Value and the Price of an Asset

A financial security is something that entitles the holder to periodic payments (cash flows)

The classical theory of asset prices is that the price of an asset is equal to the present discounted value of all future cash flows

The price of a bond (or any asset) is just the present discounted value of cash flows

The value of a shopping cart full of groceries is just the sum of the values of different goods in the cart

A financial security is the same idea, except the "different goods in the cart" are cash flows at different points in the future

Price and Yield

The yield to maturity (or just yield) is the discount rate you use to discount future cash flows

The yield is also equal to the expected or required return if you held the asset to maturity

- Not necessarily equal to realized return if security is sold prior to maturity

Key issue in asset pricing: how do you determine the yield to use to discount cash flows and hence price an asset?

Basic gist: the riskier the future cash flows are, the higher the required return you demand to hold an asset, and therefore the more you discount future cash flows

Different Types of Bonds

1. Simple loan: you borrow X dollars and agree to pay back $(1+i)X$ dollars at some specified date (interest plus principal) (e.g., commercial loan)

2. Fixed payment loan: you borrow X dollars and agree to pay back the same amount each period (e.g. month) for a specified period of time. "Full amortization" (e.g., fixed rate mortgage)

3. Coupon bond: you borrow X dollars and agree to pay back fixed coupon payments, C, each period (e.g. year) for a specified period of time (e.g. 10 years), at which time you pay off the "face value" of the bond (e.g., Treasury Bond)

4. Discount bond: you borrow X dollars and agree to pay back Y dollars after a specified period of time with no payments in the intervening periods. Typically, $Y > X$, so the bond sells "at a discount" (e.g., Treasury Bill)

Yield to Maturity

The yield to maturity (YTM) is the (fixed) interest rate that equates the PDV of cash flows with the price of the bond in the present

This measures the return (expressed at an annualized rate) that would be earned on holding a bond if it is held until maturity (and there is no default, so more accurately this is an expected return)

The YTM does not necessarily correspond to the realized return if the bond is not held until maturity (i.e., if you sell a bond before its maturity date)

The YTM is another way of conveying the price of a bond, taking future cash flows as given

- Price depends on size of cash flows (i.e., face value); yield does not

YTM on a Simple Loan

The YTM on a simple loan is just the contractual interest rate

Let P be the price of the loan, CF the payout after one year, and i the interest rate. Then:

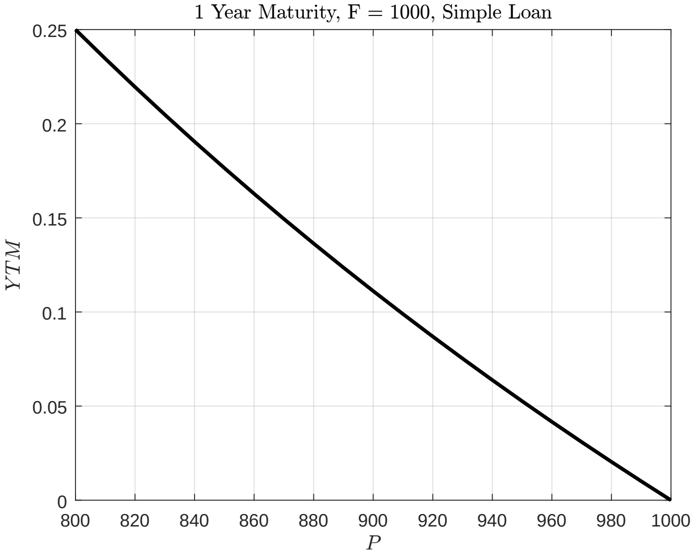
$$P = \frac{CF}{1+i}$$

$$1 + i = \frac{CF}{P}$$

$$i = \frac{CF - P}{P}$$

Where $\frac{CF-P}{P}$ is the return (or rate of return) on the loan

Price and Yield on a Simple Loan



YTM on a Discount Bond

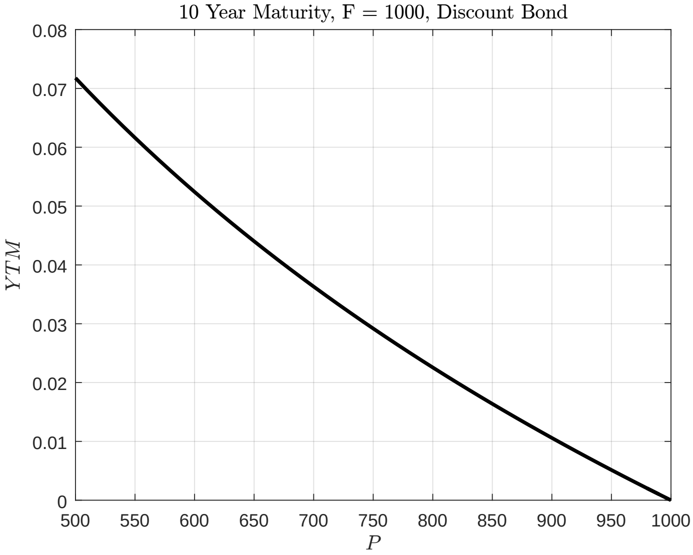
The YTM on a discount bond is similar to a simple loan, just with different time to maturity

In particular, for a face value of F, maturity of n, and price of P, the YTM satisfies:

$$P = \frac{F}{(1+i)^n}$$

$$1 + i = \left(\frac{F}{P}\right)^{\frac{1}{n}}$$

Price and Yield on a Discount Bond



Coupon Bond

Suppose a bond has a face value of \$100 and a maturity of three years

It pays coupon payments of \$10 in years $t+1$, $t+2$, and $t+3$ (the coupon rate in this example is 10 percent)

The face value is paid off after period $t+3$

Price and Yield on a Coupon Bond

For a coupon payment of C and n period maturity, price and yield satisfy:

$$P = \sum_{j=1}^{n} \frac{C}{(1+i)^j} + \frac{FV}{(1+i)^n}$$

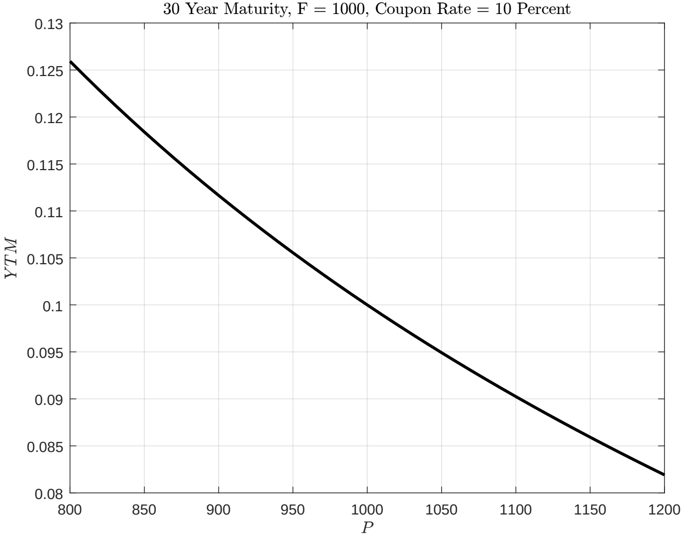
Knowing the price tells you the yield and vice-versa

In example above, suppose the period t price of the bond is \$100. The YTM solves:

$$100 = \frac{10}{1+i} + \frac{10}{(1+i)^2} + \frac{10}{(1+i)^3} + \frac{100}{(1+i)^3}$$

Which works out to $i = 0.1$

Price and Yield on a Coupon Bond



Coupon Rate vs. Yield

In the previous example, the coupon rate (10 percent) and yield (10 percent) are the same

In general, this is not going to be true

Will only be true if the bond "sells at par" – meaning the price of the bond equals the face value. If bond sells at a discount to face value, yield is greater than coupon rate and vice-versa

Coupon rate is not the same thing as "the" interest rate!

Perpetuities

Perpetuities (also called "consols") are like coupon bonds, except they have no maturity date

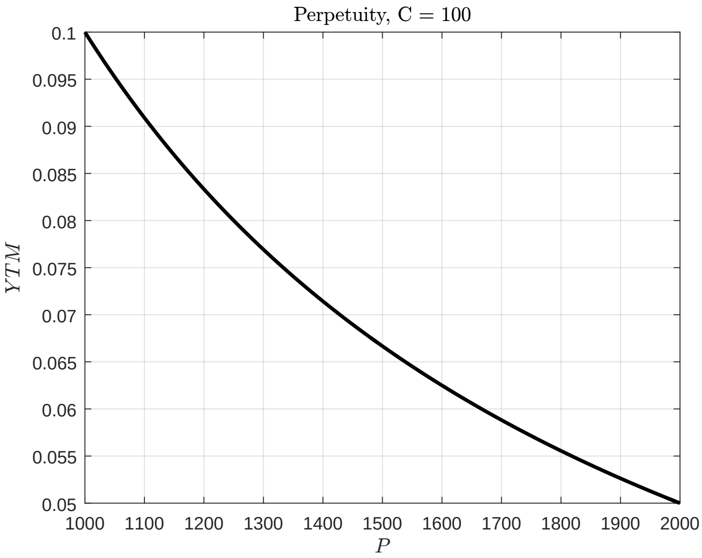
Here, the relationship between price, yield, and coupon payments works out cleanly and is given by:

$$i = \frac{C}{P}$$

For a coupon bond with a sufficiently long maturity, this is a reasonable approximation to the bond's YTM (because the PDV of the face value after many years is close to zero)

This expression is also sometimes called the current yield as an approximation to the YTM on a coupon bond

Price and Yield on a Perpetuity



Observations

1. The bond price and yield are negatively related. This is true for all types of bonds. **Bond prices and interest rates move in opposite directions**
2. For discount bonds, we would not expect price to be greater than face value – this would imply a negative yield
3. For a coupon bond, when the bond is priced at face value, the yield to maturity equals the coupon rate
4. For a coupon bond, when the bond is priced less than face value, the YTM is greater than the coupon rate (and vice-versa
5. Discount bond is a special case of a coupon bond with coupon rate of zero
6. Perpetuity is a special case of a coupon bond with no maturity (and hence no face value)

Yields (Interest Rates) and Returns

Realized returns and yields (interest rates) are in general not the same thing

Rate of return: cash flow plus new security price, divided by current price

Useful way to think about it (terminology here is related to equities): "dividend rate plus capital gain," where capital gain is the change in the security's price

The return on a coupon bond held from t to $t+1$ is:

$$R = \frac{C + P_{t+1} - P_t}{P_t}$$

$$R = \underbrace{\frac{C}{P_t}}_{\text{Current Yield}} + \underbrace{\frac{P_{t+1} - P_t}{P_t}}_{\text{Capital Gain}}$$

Discount Bond

Suppose that you hold a discount bond with face value \$1000, a maturity of 30 years, and a current yield to maturity of 10 percent. The current price of this bond is $\frac{1000}{1.1^{30}} = 57.31$

Suppose that interest rates stay the same after a year. Then the bond has a price of $\frac{1000}{1.1^{29}} = 63.04$

Since there is no coupon payment, your one-year holding period return (holding period refers to length of time you hold the security) is just the capital gain:

$$R = \frac{P_{t+1} - P_t}{P_t} = \frac{63.04 - 57.31}{57.31} = 0.10$$

If interest rates do not change, then the realized return and the yield to maturity are the same thing

Interest Rate Risk

But now suppose that interest rates go up to 15 percent in period $t+1$ and are expected to remain there

Then the price of the bond in period $t+1$ will be: $\frac{1000}{1.15^{29}} = 17.37$ – i.e. the bond price falls Your return is then:

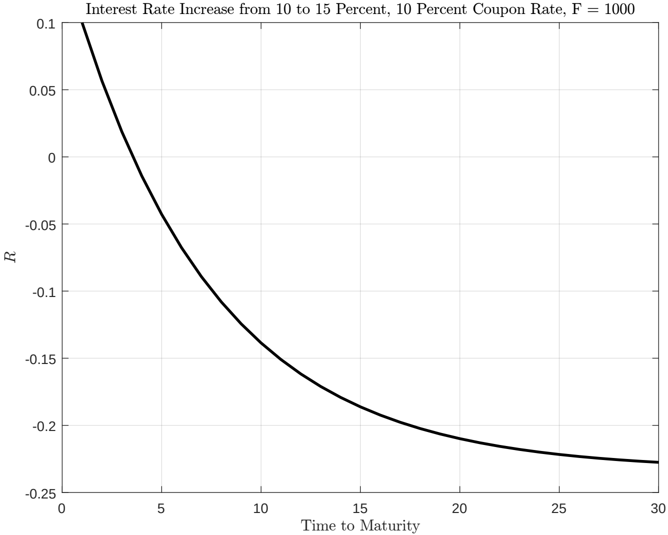
$$R = \frac{P_{t+1} - P_t}{P_t} = \frac{17.37 - 57.31}{57.31} = -0.69$$

On a discount bond, an increase in interest rates exposes you to capital loss

True more generally for a coupon bond – while you know current yield at time of investment, capital gain is unknown

What we call interest rate risk (closely related concept is duration risk)

Holding Period Return and Time to Maturity, Coupon Bond



Observations

Return and initial YTM are equal if the holding period is the same as time to maturity (1 period)

Increase in interest rates results in returns being less than yield (and vice-versa)

Return is more affected by interest rate change the longer is the time to maturity

If you hold the bond until maturity, your return is locked in at initial YTM

Concept of return is relevant even if you do not sell the bond and realize the capital loss. There is an opportunity cost – if interest rates rise, had you not locked yourself in on a long maturity bond you could have purchased a bond in the future with a higher yield

Longer maturity bonds are therefore <u>riskier</u> than short maturity bonds

Determinants of Bond Prices (and Interest Rates)

We've talked about specifics of bonds

But what determines prices (and hence yields)?

Two related approaches:

- Conceptual (Mishkin): demand and supply
- Micro-founded (GLS): explicit consumption-saving maximization problem

Conceptual Approach: Portfolio Choice

A bond (or any asset) is a way to transfer resources intertemporally

Demand for a bond is based on the following factors:

1. Wealth/income: assets are normal goods, so the more wealth, the more you want to hold at every price
2. Expected returns: you hold assets to earn returns. The higher the expected return, the more of it you demand
3. Risk: assume agents are risk averse. Holding expected return constant, you would prefer a less risky return. The more risky an asset is, the less of it you demand
4. Liquidity: refers to the ease with which you can sell an asset without affecting its price. The more liquid it is, the more attractive it is to hold (it's easier to sell at a known price if you need to raise cash in a pinch)

Demand for Bonds

How does the demand for bonds vary with the price of bonds?

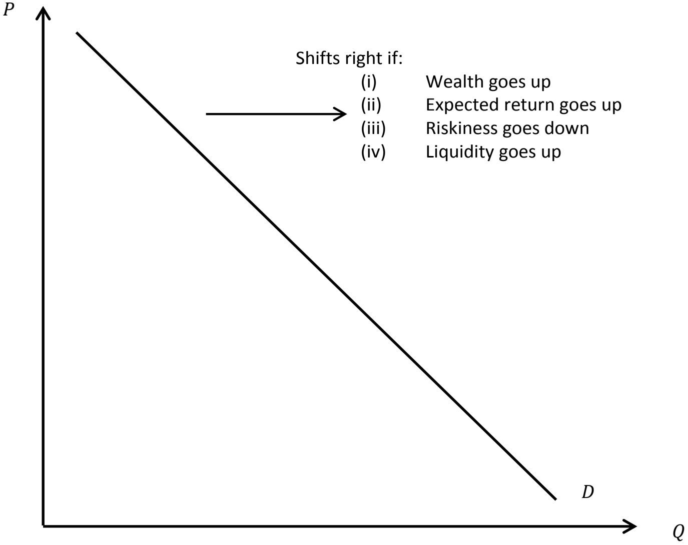
As the price goes down, the yield goes up

Therefore, holding everything else fixed, the expected return (yield) on holding a bond goes up as the price falls

Therefore, demand slopes down

Demand will shift (change in quantity demanded for a given price) with changes in other factors

Bond Demand



P
Shifts right if:
(i) Wealth goes up
(ii) Expected return goes up
(iii) Riskiness goes down
(iv) Liquidity goes up
D
Q

Supply of Bonds

Issuers of bonds are borrowers. You are issuing a bond to raise funds in the present to be paid back in the future

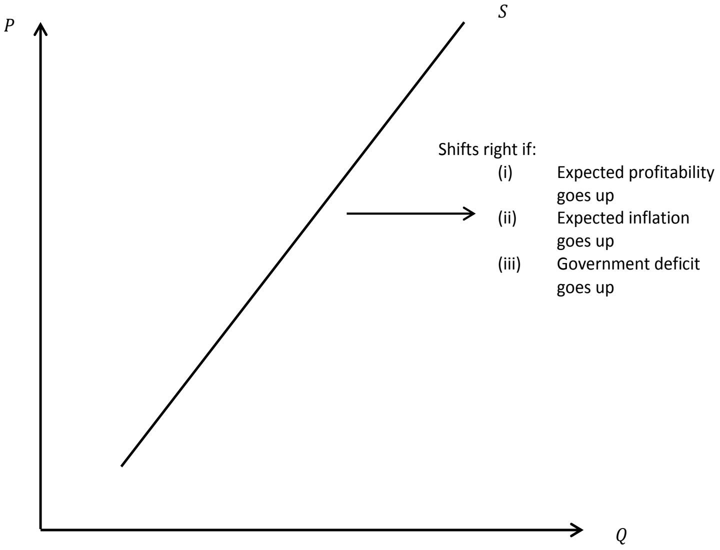
As the bond price increases, the yield decreases

Therefore, at a higher bond price, the cost of borrowing is lower

So there will be more supply of bonds at a higher price – supply slopes up

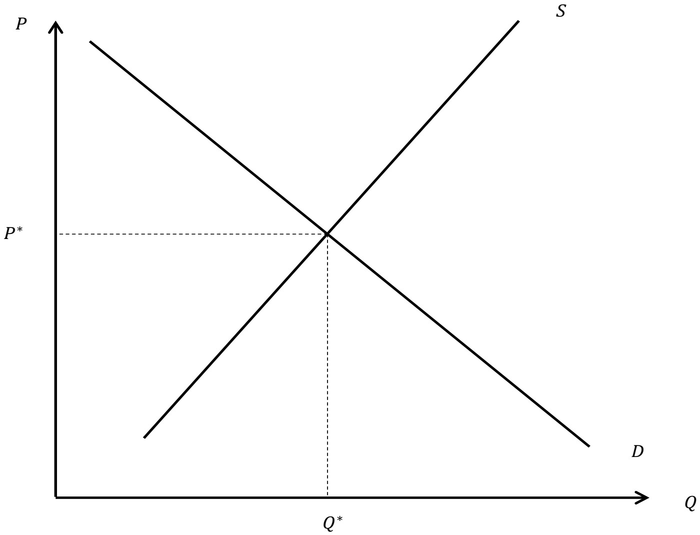
Changes in other factors, holding price fixed, will cause the supply curve to shift

Bond Supply

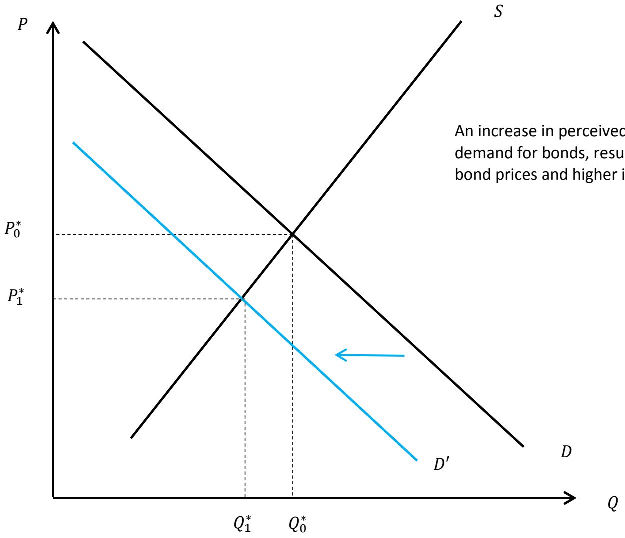


P
S
Q
Shifts right if:
(i) Expected profitability goes up
(ii) Expected inflation goes up
(iii) Government deficit goes up

Bond Market Equilibrium

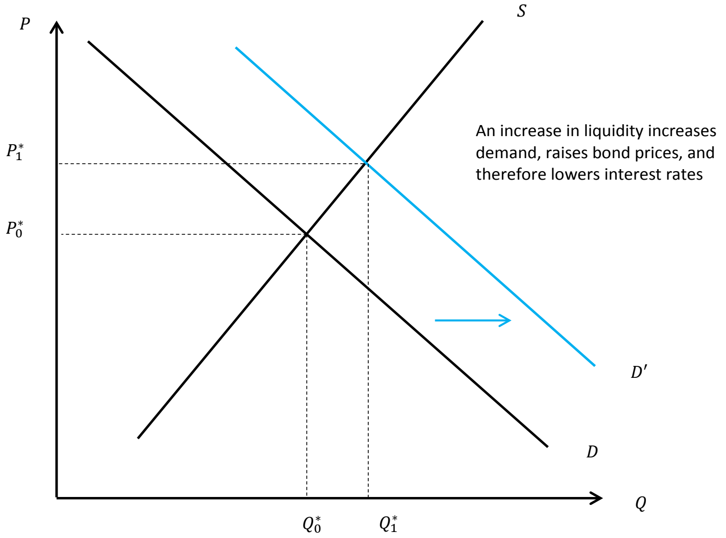


An Increase in Risk

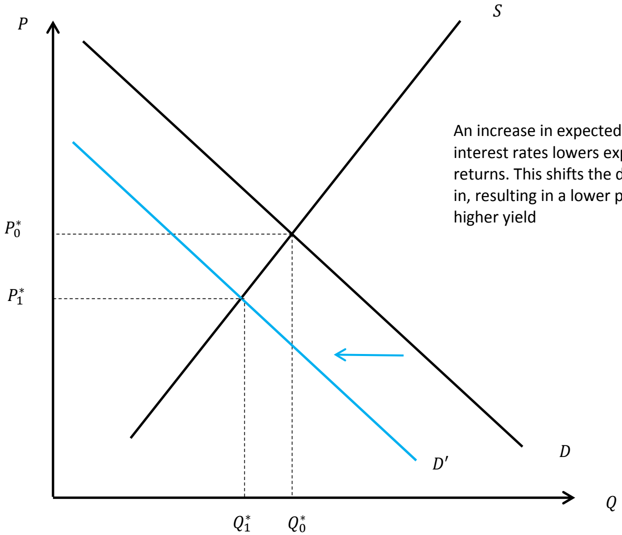


An increase in perceived risk reduces demand for bonds, resulting in lower bond prices and higher interest rates

An Increase in Liquidity

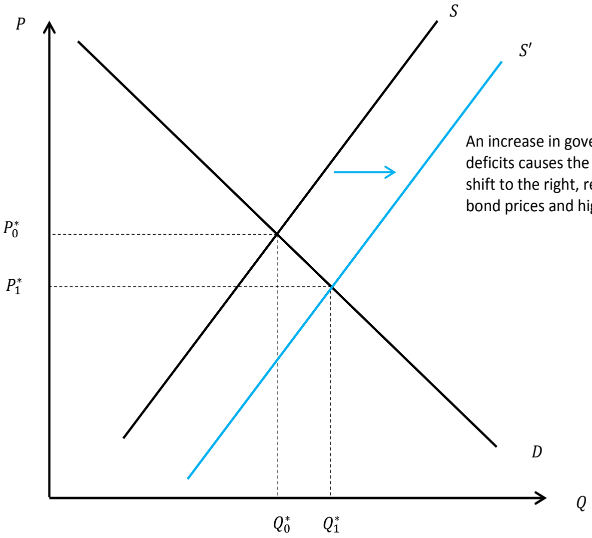


An Increase in Expected Future Interest Rates



An increase in expected future interest rates lowers expected bond returns. This shifts the demand curve in, resulting in a lower price and higher yield

An Increase in Government Budget Deficit



An increase in government budget deficits causes the supply of bonds to shift to the right, resulting in lower bond prices and higher interest rates

Micro-Founded Approach

Household lives (at least) two periods: t and $t+1$

Can purchase/sell one-period discount bonds at P_t^B. Face value is one

Assume no uncertainty. Lifetime utility:

$$U = u(C_t) + \beta u(C_{t+1})$$

Budget constraints:

$$C_t + P_t^B B_t = Y_t$$

$$C_{t+1} = Y_{t+1} + B_t$$

Euler Equation

Optimization yields an Euler equation:

$$P_t^B = \frac{\beta u'(C_{t+1})}{u'(C_t)}$$

Since $P_t^B = (1 + r_t)^{-1}$, where r_t is the yield, this can also be written as in intermediate macro:

$$u'(C_t) = \beta u'(C_{t+1})(1 + r_t)$$

Marginal benefit = marginal cost interpretation

Stochastic Fiscount Factor

The term $\frac{\beta u'(C_{t+1})}{u'(C_t)}$ is called the stochastic discount factor, $m_{t,t+1}$

Really just a fancy term for marginal rate of substitution (MRS)

General asset pricing condition:

$$P_t^A = \mathbb{E}\left[\sum_{j=1}^{\infty} m_{t,t+j} D_{t+j}^A\right]$$

Where $\mathbb{E}$ is expectations operator (allows for uncertainty over future), A indexes an asset, D_{t+j}^A is the cash flow generated by the asset in period $t+j$, and $m_{t,t+j} = \frac{\beta^j u'(C_{t+j})}{u'(C_t)}$ is the stochastic discount factor, and the summation allows security to pay cash flows in multiple periods (so more general than a discount bond)

Prices and Yields

Yield on one period discount bond:

$$1 + i_{B,t} = \frac{1}{P_t^B}$$

Useful to compare assets according to yields instead of prices. Why? Suppose you have a discount bond that pays out 2 in $t+1$ (label this D) but is otherwise identical to B

Price of asset D will be double price of B. But yields are identical:

$$1 + i_{B,t} = 1 + i_{D,t} = \frac{1}{P_t^B} = \frac{2}{P_t^D}$$

Comparing yields puts assets with different levels of cash flow on "equal footing"

Different Bond Characteristics

Bonds with the same cash flow details (e.g., discount bonds) nevertheless often have very different yields

Aside from details about cash flows, bonds differ principally on two dimensions:

1. Default risk
2. Time to maturity

The first, and potentially the second, generate <u>uncertainty</u>

People don't like uncertainty: more uncertainty $\Rightarrow$ lowers bond price (raises yield)

Default Risk

Default occurs when the borrower decides to not make good on a promise to pay back all or some of his/her outstanding debts

We think of federal government bonds as being (essentially) default-free: since government can always "print" money, should not explicitly default (though monetization of debt is effectively form of default)

Corporate (and state and local government) bonds do have some default risk

Rating agencies: Aaa is highest rated, then Bs, then Cs measure credit risk of borrowers

Risk premium: difference (i.e., spread) between yield on relatively more risky debt (e.g., Aaa corporate debt) and less risk debt (e.g., government debt), assuming same time to maturity

Yields on Different Bonds

Observations

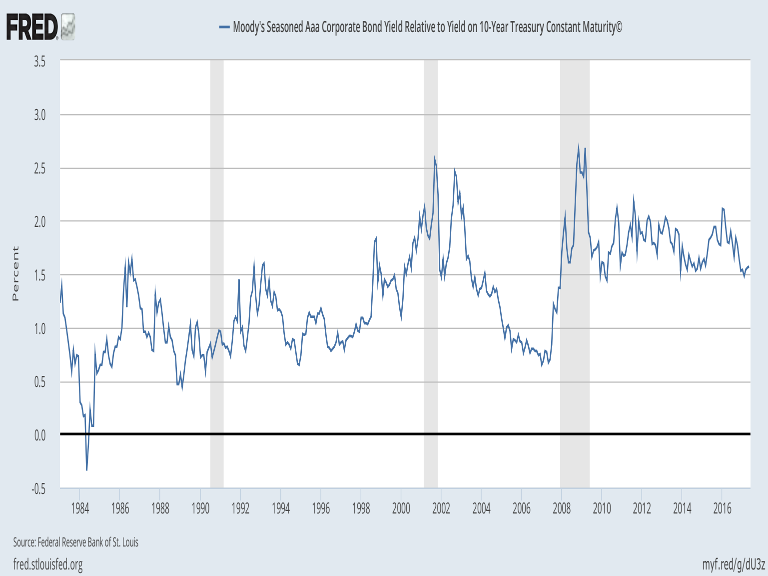
We see more or less exactly the pattern we would expect from our demand/supply analysis – riskier bonds have higher yields

You demand a higher yield (expected return) to be compensated for taking on more risk

Obvious exception: municipal bonds

Why? Interest income on these bonds is exempt from federal taxes, which makes their expected returns higher, and therefore drives up price (and drives down yield)

Important: interesting time variation in spreads (see next couple of slides)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Moody's Seasoned Aaa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity©
Percent
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
1984
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Source: Federal Reserve Bank of St. Louis
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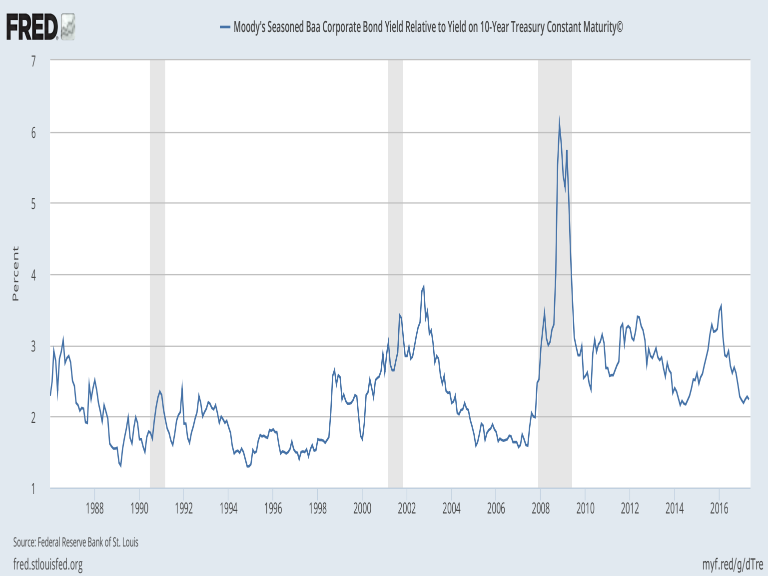
FRED
Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity©
Percent
7
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5
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1992
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Source: Federal Reserve Bank of St. Louis
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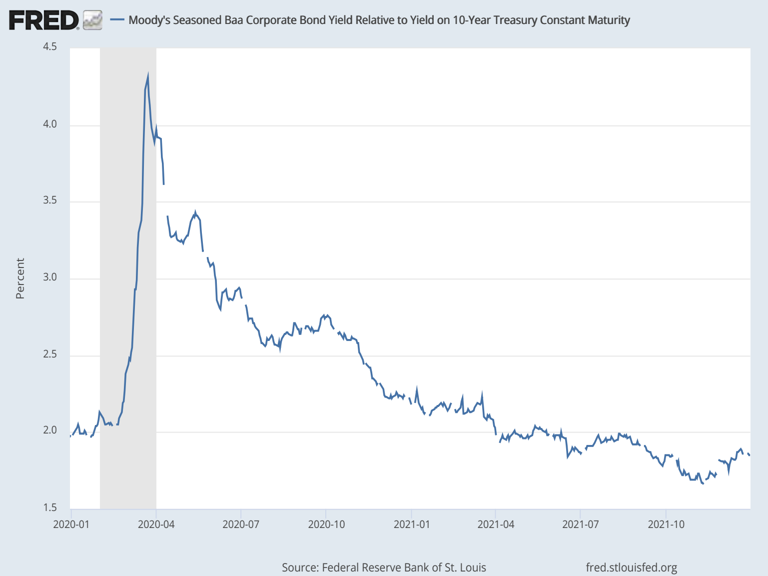

FRED
Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity
Percent
4.5
4.0
3.5
3.0
2.5
2.0
1.5
2020-01
2020-04
2020-07
2020-10
2021-01
2021-04
2021-07
2021-10
Source: Federal Reserve Bank of St. Louis
fred.stlouisfed.org

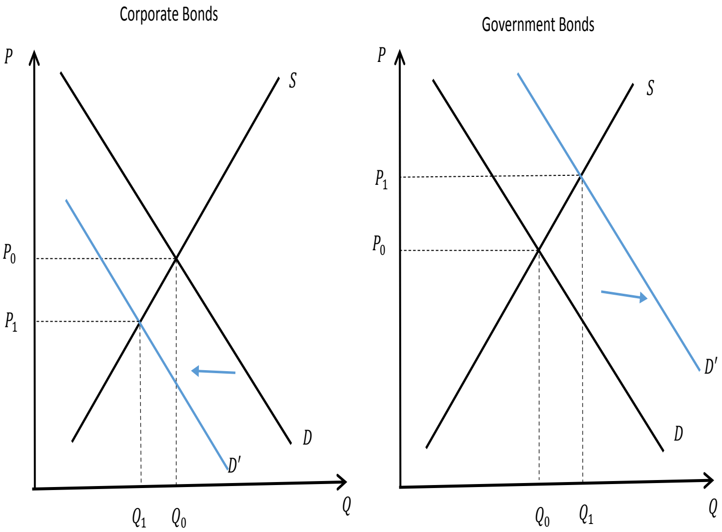
Countercyclical Default Risk

It stands to reason that default risk ought to be high when economy is in recession

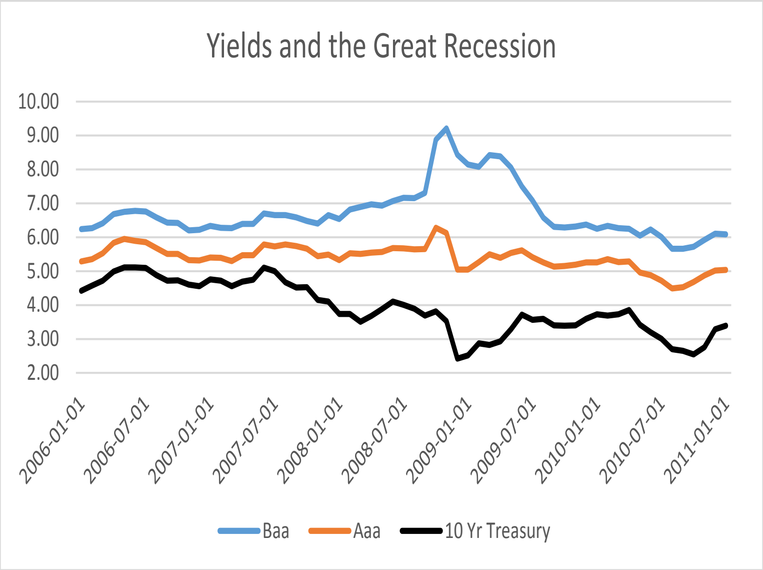
When default risk is high, we might expect a “flight to safety”: reduces demand for risky bonds and increases demand for riskless bonds, which moves credit spread up

This is consistent with what we see in the previous slides: credit spreads tend to rise during recessions

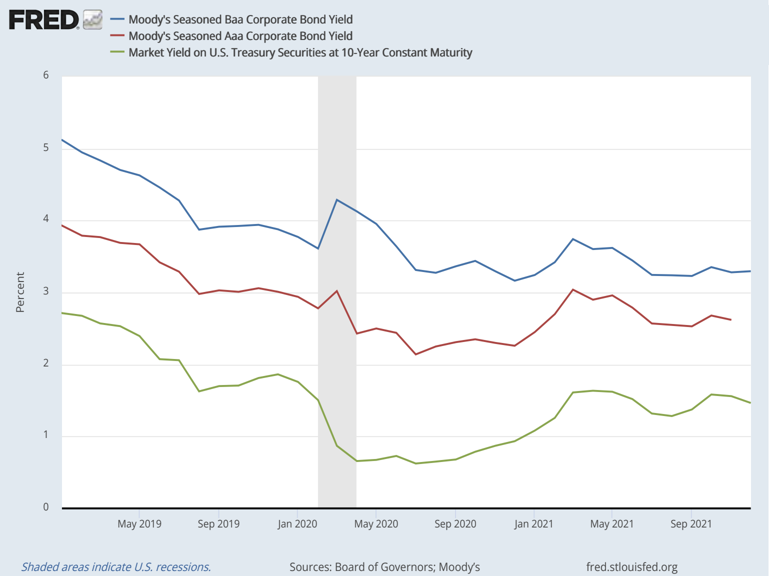
Flight to Safety



Yields During Great Recession



Yields During COVID-19



A Micro-Founded Approach

Household lives for two periods. Receives endowment of income

Can save in period t either via discount bonds $B_{1,t}$ or $B_{2,t}$. $B_{1,t}$ is risk-free. $B_{2,t}$ might default (completely, so pays back no principal)

Current endowment is known. Future endowment is uncertain. Four possible states of the world in $t+1$:

$$\begin{aligned}
\text{State 1}: &\quad Y_{t+1} = Y^h_{t+1}, \text{ risky bond pays} \\
\text{State 2}: &\quad Y_{t+1} = Y^h_{t+1}, \text{ risky bond defaults} \\
\text{State 3}: &\quad Y_{t+1} = Y^l_{t+1}, \text{ risky bond pays} \\
\text{State 4}: &\quad Y_{t+1} = Y^l_{t+1}, \text{ risky bond defaults}
\end{aligned}$$

Probabilities are p_1, p_2, p_3, and p_4

$p_1 + p_2 + p_3 + p_4 = 1.$

Expected Utility and Budget Constraints

Household wishes to maximize expected utility:

$$
\begin{aligned}
U &= u(C_t) + \beta\,\mathbb{E}[u(C_{t+1})] \\
&= u(C_t) + \beta\left[p_1 u(C_{t+1}(1)) + p_2 u(C_{t+1}(2)) + p_3 u(C_{t+1}(3)) + p_4 u(C_{t+1}(4))\right]
\end{aligned}
$$

Budget constraints must hold in each state of the world:

$$
C_t + P^B_{1,t} B_{1,t} + P^B_{2,t} B_{2,t} = Y_t
$$

$$
\begin{aligned}
C_{t+1}(1) &= Y^h_{t+1} + B_{1,t} + B_{2,t} \\
C_{t+1}(2) &= Y^h_{t+1} + B_{1,t} \\
C_{t+1}(3) &= Y^l_{t+1} + B_{1,t} + B_{2,t} \\
C_{t+1}(4) &= Y^l_{t+1} + B_{1,t}
\end{aligned}
$$

Facts About Expectations Operators

Just probability-weighted sum of outcomes:

$$\mathbb{E}[X_{t+1}] = p_1 X_{t+1}(1) + p_2 X_{t+1}(2) + \cdots + p_n X_{t+1}(n)$$

Useful facts:

$$\mathbb{E}[f(X_{t+1})] \neq f\left(\mathbb{E}[X_{t+1}]\right) \text{ unless } f(\cdot) \text{ is linear}$$

$$\mathbb{E}[aX_{t+1})] = a\,\mathbb{E}[X_{t+1}]$$

$$\mathbb{E}[X_{t+1} + Y_{t+1}] = \mathbb{E}[X_{t+1}] + \mathbb{E}[Y_{t+1}]$$

$$\frac{\partial\,\mathbb{E}[f(X_{t+1})]}{\partial X_{t+1}} = \mathbb{E}[f'(X_{t+1})]$$

$$\mathbb{E}[X_{t+1} Y_{t+1}] = \mathbb{E}[X_{t+1}]\,\mathbb{E}[Y_{t+1}] + cov(X_{t+1}, Y_{t+1})$$

$$cov(X_{t+1}, Y_{t+1}) = \mathbb{E}\left[(X_{t+1} - \mathbb{E}(X_{t+1}))\left(Y_{t+1} - \mathbb{E}(Y_{t+1})\right)\right]$$

Endowment Economy

Suppose that income is exogenously given (i.e., an endowment economy)

Consumption (in all states) can be inferred via the budget constraints

Need to know something about bond supply to “price” the two bonds

Simplifying assumption: bonds are in zero net supply, i.e., $B_{1,t} = B_{2,t} = 0$

Not necessary, but makes life easier

Bond Pricing in Endowment Economy

Risk-free bond:

$$P_{1,t} = \mathbb{E}\left[\frac{\beta u'(Y_{t+1})}{u'(Y_t)}\right]$$

Risky bond:

$$P_{2,t} = \mathbb{E}\left[\frac{\beta u'(Y_{t+1})}{u'(Y_t)} D_{2,t+1}\right]$$

Where $D_{2,t+1}$ is the payout on the risky bond (either 1 or 0).
Yields:

$$1 + i_{1,t} = \frac{1}{P^B_{1,t}} = \frac{u'(Y_t)}{\mathbb{E}[\beta u'(Y_{t+1})]}$$

$$1 + i_{2,t} = \frac{\mathbb{E}[D_{2,t+1}]}{P^B_{2,t}} = \frac{\mathbb{E}[D_{2,t+1}]u'(Y_t)}{\mathbb{E}[\beta u'(Y_{t+1})D_{2,t+1}]}$$

Risk Premium

Ratio of yields is:

$$\frac{1+i_{2,t}}{1+i_{1,t}} = \frac{\mathbb{E}[D_{2,t+1}]\,\mathbb{E}[u'(Y_{t+1})]}{\mathbb{E}[D_{2,t+1}u'(Y_{t+1})]}$$

One would be tempted to "distribute" the expectations operator and conclude this is 1 (so no difference in yields)

But in general cannot do this

$$i_{2,t} - i_{1,t} \approx -\frac{cov(D_{2,t+1}, u'(Y_{t+1}))}{\mathbb{E}[D_{2,t+1}u'(Y_{t+1})]}$$

Risk premium depends on covariance of bond payout with $u'(Y_{t+1})$, not variance of payout per se

Intuition

Assuming $u''(\cdot) < 0$ (what we call risk averse), you desire to smooth consumption

Both across time as well as across states

For assets, you like assets that have comparatively high payouts in periods where income is low (i.e., periods where $u'(Y_{t+1})$ is high, so consumption is highly valued). This helps you smooth consumption across states

Dislike assets that don't help you do this: only willing to pay a low price (i.e., demand a high expected return)

Assets most likely to default in "bad" periods (where $u'(Y_{t+1})$ is high): you demand a higher yield to hold them (lower price)

Countercyclical default risk: source of bond risk premia

Uncertainty and the Risk Premium

$Y_{t+1}^h = 1.1,\ Y_{t+1}^l = 0.9,\ Y_t = 1,\ \beta = 0.95$, log utility

Recall: state 1: income high, bond pays; state 2: income high, bond defaults; state 3, income low, bond pays; state 4: income low, bond defaults

Probabilities	$\mathbb{E}(D)$	$\mathbb{E}(Y_{t+1})$	$\mathbb{E}(D_{2,t+1} \mid Y_{t+1}^h)$	$\mathbb{E}(D_{2,t+1} \mid Y_{t+1}^l)$	$i_{2,t} - i_{1,t}$
$p_1 = p_2 = p_3 = p_4 = 0.25$	0.5	1	0.5	0.5	0.00
$p_1 = 0.5,\ p_2 = p_3 = 0,\ p_4 = 0.5$	0.5	1	1	0	0.12
$p_1 = p_4 = 0,\ p_2 = p_3 = 0.5$	0.5	1	0	1	-0.09
$p_1 = 0.4,\ p_2 = 0.1,\ p_3 = 0.1,\ p_4 = 0.4$	0.5	1	0.8	0.2	0.07

If default is mostly likely in periods where income is low, you demand a higher yield relative to the risk-free bond

In principle, risk premium could be negative if default is most likely when income is high

Also, risk premium doesn't depend on uncertainty over default per se

Term Structure of Interest Rates

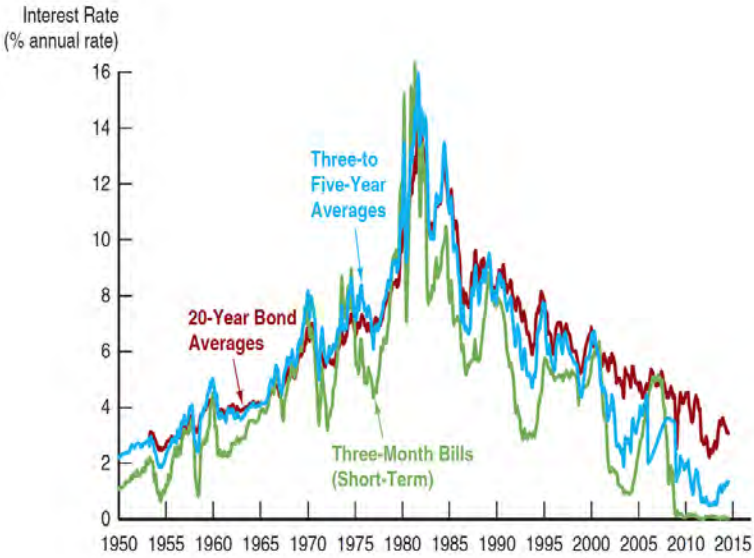
Bonds with otherwise identical cash flows and risk characteristics can have different times to maturity

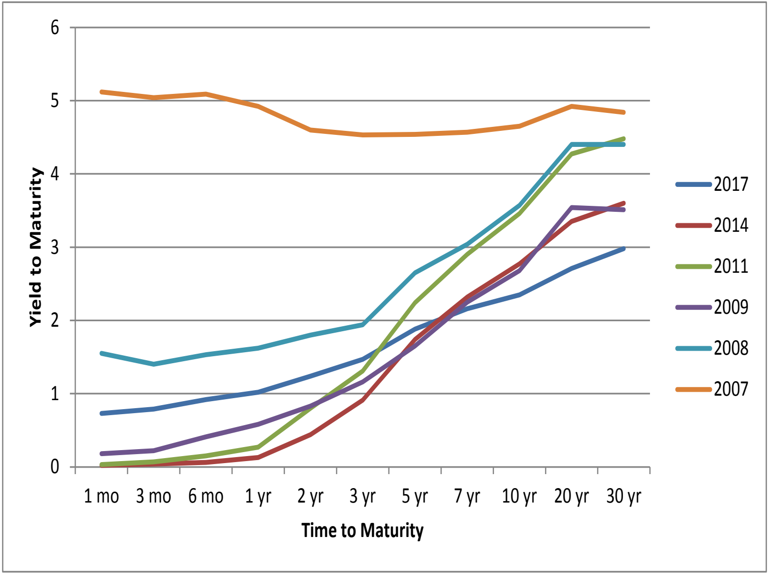
Think about (default) riskless Treasury securities with different maturities

- Treasury bills: discount bonds with maturities of 4, 13 (three month T-Bill), 26, or 52 week maturities
- Treasury notes: coupon bonds with maturities of 2, 3, 5, 7 and 10 years
- Treasury bonds: coupon bonds with maturities $>$ 10 years

How do yields vary with time to maturity for a bond with otherwise identical default characteristics?

A plot of yields on bonds against the time to maturity is known as a yield curve


Interest Rate
(% annual rate)
16
14
12
10
8
6
4
2
0
1950 1955 1960 1965 1970 1975 1980 1985 1990 1995 2000 2005 2010 2015
Three-to
Five-Year
Averages
20-Year Bond
Averages
Three-Month Bills
(Short-Term)

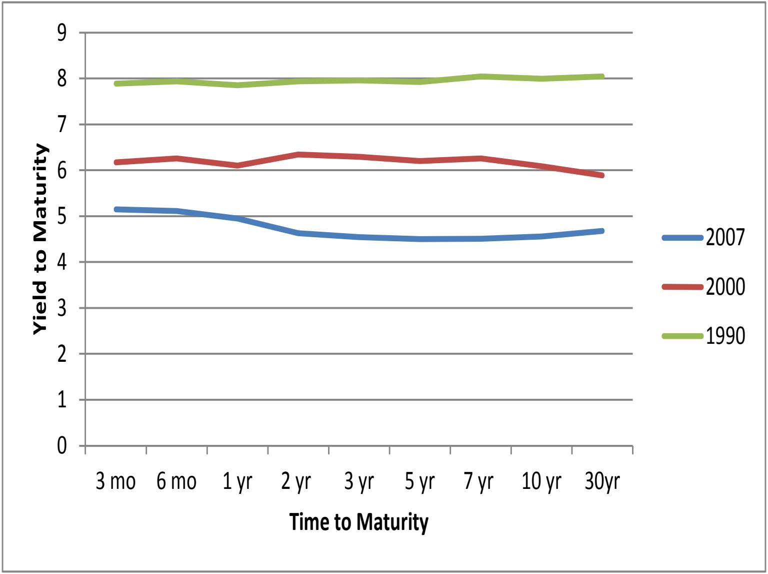

6
5
4
3
2
1
0
Yield to Maturity
1 mo
3 mo
6 mo
1 yr
2 yr
3 yr
5 yr
7 yr
10 yr
20 yr
30 yr
Time to Maturity
2017
2014
2011
2009
2008
2007

Observations

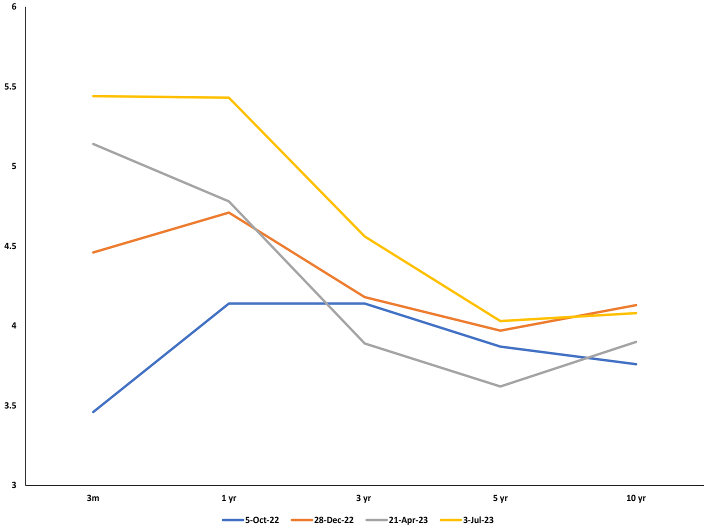
The following observations can be made from previous two pictures

1. Yields on bonds of different maturities tend to move together
2. Yield curves are upward-sloping *most of the time*
   - The slope of the yield curve is often predictive of recession. Flat or downward-sloping ("inverted") yield curves often precede recessions
   - But not always!
3. When short term interest rates are low, yield curves are more likely to be upward-sloping

Yield Curves Prior to Recent Recessions



Post-Covid Yield Curves



Theories of the Yield Curve

Main theories of the term structure:

1. Expectations hypothesis
2. Segmented markets
3. Liquidity premium theory

Theories of the Yield Curve Explained

Expectations hypothesis: bonds of different maturities are perfect substitutes

Segmented markets: bonds with different maturities are not substitutes

Liquidity premium: bonds with different maturities are imperfect substitutes

- Essentially expectations hypothesis + uncertainty, where longer maturity bonds are riskier than short maturity bonds because their prices fluctuate more

Expectations Hypothesis

Expectations hypothesis: the yield on a long maturity bond is the average of the expected yields on shorter maturity bonds

For example, suppose you consider 1 and 2 year bonds

- The yield on a 1 year bond is 4 percent; you expect the yield on a 1 year bond one year from now to be 6 percent
- Then the yield on a two year bond ought to be 5 percent $(0.5 \times (4+6) = 5)$
- Why? If you buy a two 1 year bonds in succession, your yield over the two year period is (approximately, ignoring compounding) 10 percent: $(1+0.04) \times (1+0.06) - 1 = 0.1024 \approx 0.10$
- If the 2 and 1 year bonds are perfect substitutes, the yield on the two year bond has to be the same: $(1+i)^2 = 0.1 \Rightarrow i \approx 0.05$

Microfoundations of Expectations Hypothesis

Suppose a household lives for three periods, receiving an exogenous endowment of income in each period

In period t, can purchase one- or two-period maturity discount bonds:

$$C_t + P_{t,t,t+1}B_{t,t,t+1} + P_{t,t,t+2}B_{t,t,t+2} = Y_t$$

Subscript notation. First subscript denotes date of observation (i.e. period t is the "present"). Second subscript denotes date of issuance, third subscript denotes date of maturity. So $B_{t,t,t+1}$ is the quantity of a bond purchased in period t (first subscript), that was issued in period t (second subscript), which matures in $t+1$ (third subscript)

Future Budget Constraints

Assume, for now, no uncertainty over the future

In $t+1$, household can buy/sell newly issued one period bonds or change its holdings of existing two period bonds (no new issues of two period bonds since there is no $t+3$). Receives exogenous endowment income plus maturing one-period bonds

$$C_{t+1} + P_{t+1,t+1,t+2}B_{t+1,t+1,t+2} + P_{t+1,t,t+2}(B_{t+1,t,t+2} - B_{t,t,t+2}) = Y_{t+1} + B_{t,t,t+1}$$

$B_{t+1,t,t+2} - B_{t,t,t+2}$: change in stock of two-period bonds (can buy/sell on secondary market)

In final period, just consume available resources:

$$C_{t+2} = Y_{t+2} + B_{t+1,t+1,t+2} + B_{t+1,t,t+2}$$

Preferences and Optimality Conditions

Lifetime utility:

$$U = \ln C_t + \beta \ln C_{t+1} + \beta^2 \ln C_{t+2}$$

Optimality conditions:

$$\begin{aligned}
B_{t,t,t+1} &: \; P_{t,t,t+1} u'(C_t) = \beta u'(C_{t+1}) \\
B_{t,t,t+2} &: \; P_{t,t,t+2} u'(C_t) = \beta P_{t+1,t,t+2} u'(C_{t+1}) \\
B_{t+1,t+1,t+2} &: \; P_{t+1,t+1,t+2} u'(C_{t+1}) = \beta u'(C_{t+2}) \\
B_{t+1,t,t+2} &: \; P_{t+1,t,t+2} u'(C_{t+1}) = \beta u'(C_{t+2})
\end{aligned}$$

Note: must have $P_{t+1,t,t+2} = P_{t+1,t+1,t+2}$

Price of bond depends on <u>time to maturity</u> not <u>date of issuance</u>

Combining These Together

If we combine these FOC, we get:

$$P_{t,t,t+2} = P_{t,t,t+1} \times P_{t+1,t+1,t+2}$$

Price of long bond is <u>product</u> of short maturity bond prices

Prices to Yields

Yield is discount rate which equates bond price to present discounted value of cash flows:

$$\frac{1}{(1+i_{t,t+2})^2} = \frac{1}{1+i_{t,t+1}} \frac{1}{1+i_{t+1,t+2}}$$

Note: don't need to keep track of issuance date, just date of observation and maturity date

Approximately:

$$i_{t,t+2} \approx \frac{1}{2}\left[i_{t,t+1} + i_{t+1,t+2}\right]$$

Extends to n period maturity:

$$i_{t,t+n} \approx \frac{1}{n}\left[i_{t,t+1} + i_{t+1,t+2} + \cdots + i_{t+n-1,t+n}\right]$$

Forward Rates

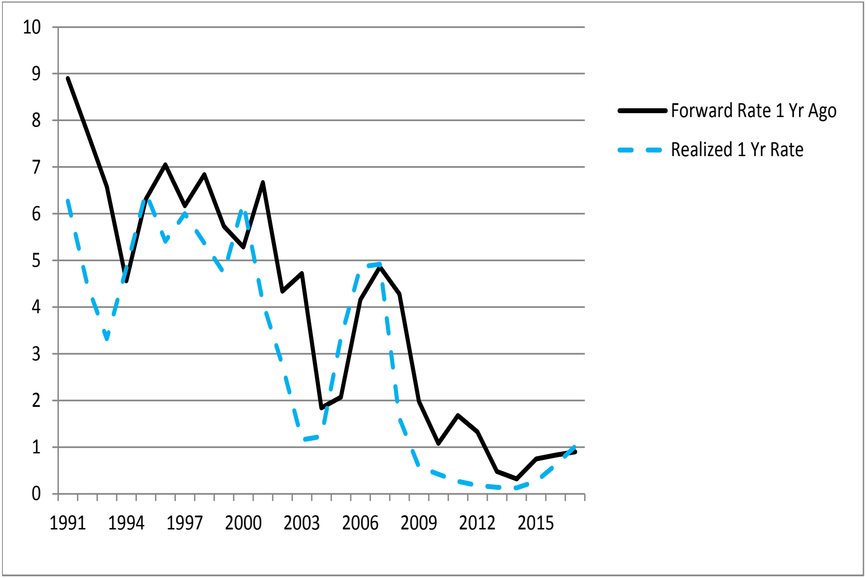
Expectations hypothesis can be used to infer market expectations of future short maturity interest rates

How? By using present yields of bonds on different maturities

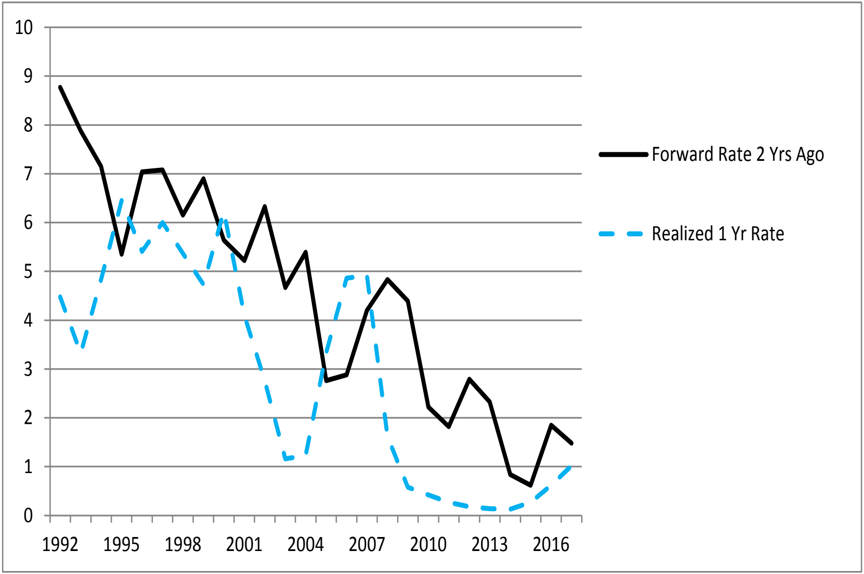
We call these forward rates:

$$
\begin{aligned}
i^e_{t+1,t+2} &= 2i_{t,t+2} - i_{t,t+1} \\
i^e_{t+2,t+3} &= 3i_{t,t+3} - 2i_{t,t+2} \\
&\vdots \\
i^e_{t+n-1,t+n} &= ni_{t,t+n} - (n-1)i_{t,t+n-1}
\end{aligned}
$$

Implied 1 Year Ahead Forward Rate



Implied 2 Year Ahead Forward Rate



Evaluating the Expectations Hypothesis

Expectations hypothesis can help make sense of several facts:

1. Long and short term rates tend to move together. If current short term rates are low and people expect them to stay low for a while (interest rates are quite persistent), then long term yields will also be low
2. Why a flattening/inversion of yield curve can predict recessions. If people expect economy to go into recession, they will expect the Fed to lower short-term interest rates in the future. If short-term interest rates are expected to fall, then long-term rates will fall relative to current short-term rates, and the yield curve will flatten.
3. Post-COVID yield curves: inflation (and interest rates) were high, but people expected inflation (and interest rates to fall). Not because of recession, but due to taming inflation

Where the expectations hypothesis fails: why are yield curves almost always upward-sloping?

Segmented Markets

Segmented markets: household heterogeneity

Some households like long-maturity debt, others prefer short-maturity debt

- e.g., investors match maturity to their retirement horizon

This could result in higher yields on long-term bonds in equilibrium (or it could not)

But because there is no relation between short- and long-term yields, it fails to make sense of other facts about the term structure

Liquidity Premium Theory

Liquidity premium theory: essentially expectations hypothesis with uncertainty (over income)

Bonds of different maturities are substitutes, but not perfect substitutes

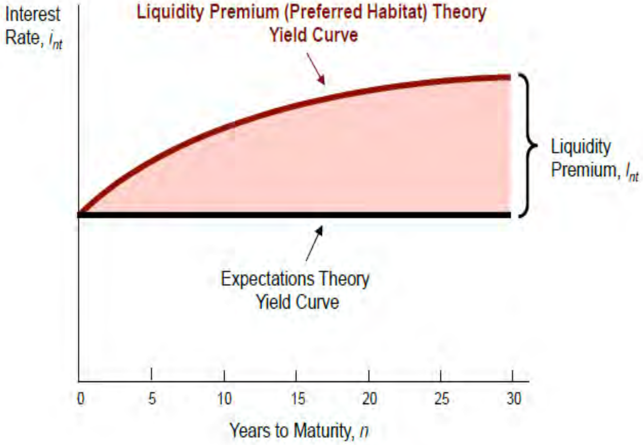
Why? Long maturity bonds are riskier – they bear interest rate risk when bond prices fluctuate

Savers demand compensation for this risk

Relationship between long and short rates:

$$i_{t,t+n} = \underbrace{\frac{i_{t,t+1} + i^e_{t+1,t+2} + \dots i^e_{t+n-1,t+n}}{n}}_{\text{Expectations Hypothesis}} + \underbrace{l_{n,t}}_{\text{Liquidity Premium}}$$

The liquidity premium, $l_{n,t}$ (also called term premium), is increasing in the time to maturity


Interest
Rate, i_{nt}
Liquidity Premium (Preferred Habitat) Theory
Yield Curve
Liquidity
Premium, l_{nt}
Expectations Theory
Yield Curve
0
5
10
15
20
25
30
Years to Maturity, n

Microfoundations of Liquidity Premium

Same setup as before, endowment economy, bonds in zero net supply, except future endowments are uncertain (still no default risk)

Endowment in $t+1$ or $t+2$ can be high or low – $Y_{t+1} = Y_{t+1}^h$ or Y_{t+1}^l (and similarly for Y_{t+2})

Assume probability is p of high income draw and $1-p$ of low endowment; same in $t+1$ and $t+2$

Budget constraints (significantly) more complicated – two states of the world in $t+1$, and essentially four states of the world in $t+2$

- Why four in $t+2$? Because consumption in $t+2$ depends on decisions made in $t+1$. So $2 \times 2 = 4$.

Objective function is to maximize expected lifetime utility We will skip a bunch of (nasty) steps but it is laid out for you in GLS Ch. 34

Optimality Conditions

Optimality conditions look the <u>same</u> as above, but expectations operators

Once again, price of bonds in $t+1$ depends only on time to maturity, not date of issuance

Using these facts, we get:

$$P_{t,t,t+1} u'(C_t) = \beta\, \mathbb{E}[u'(C_{t+1})]$$
$$P_{t,t,t+2} u'(C_t) = \beta\, \mathbb{E}[P_{t+1,t+1,t+2} u'(C_{t+1})]$$

Intuitive MB = MC interpretation, just with expectations operators

Combining These Conditions

Divide second FOC by first and re-arrange to get:

$$P_{t,t,t+2} = P_{t,t,t+1} \frac{\mathbb{E}[P_{t+1,t+1,t+2} u'(C_{t+1})]}{\mathbb{E}[u'(C_{t+1})]}$$

One is tempted to "distribute" the expectations operator in the numerator, which would yield:

$$P_{t,t,t+2} = P_{t,t,t+1} \, \mathbb{E}[P_{t+1,t+1,t+2}]$$

This would be the natural analog of what we just had, allowing for uncertainty over the future bond price

But in general you cannot do this unless (i) $u'(C_{t+1})$ is a constant (i.e. $u(\cdot)$ is linear) or (ii) $P_{t+1,t+1,t+2}$ is uncorrelated with $u'(C_{t+1})$

Re-Arranging

Can write this expression as:

$$P_{t,t,t+2} = P_{t,t,t+1}\,\mathbb{E}[P_{t+1,t+1,t+2}]\frac{\mathbb{E}[P_{t+1,t+1,t+2}u'(C_{t+1})]}{\mathbb{E}[P_{t+1,t+1,t+2}]\,\mathbb{E}[u'(C_{t+1})]}$$

If income equals consumption (bonds in zero net supply), we get:

$$P_{t,t,t+2} = \underbrace{P_{t,t,t+1}\,\mathbb{E}[P_{t+1,t+1,t+2}]}_{\text{Expectations Hypothesis}}\underbrace{\left(1+\frac{cov(P_{t+1,t+1,t+2},u'(Y_{t+1}))}{\mathbb{E}[P_{t+1,t+1,t+2}]\,\mathbb{E}[u'(Y_{t+1})]}\right)}_{\text{Term Premium}}$$

We would expect covariance term to be negative

When Y_{t+1} is low, $u'(Y_{t+1})$ is high and demand for one period bonds is low (people don't want to save), so $P_{t+1,t+1,t+2}$ is low

Negative covariance $\Rightarrow$ two-period bond trades at a discount

Prices to Yields

Written in terms of yields, approximately we get:

$$(1 + i_{t,t+2})^2 = (1 + i_{t,t+1})\,\mathbb{E}[1 + i_{t+1,t+2}]\left(1 + \frac{cov(P_{t+1,t+1,t+2}, u'(Y_{t+1}))}{\mathbb{E}[P_{t+1,t+1,t+2}]\,\mathbb{E}[u'(Y_{t+1})]}\right)^{-1}$$

Which is approximately:

$$i_{t,t+2} \approx \frac{1}{2}\left[i_{t,t+1} + i^{e}_{t+1,t+2}\right] + \frac{1}{2}tp_t$$

$$tp_t = -\ln\left(1 + \frac{cov(P_{t+1,t+1,t+2}, u'(Y_{t+1}))}{\mathbb{E}[P_{t+1,t+1,t+2}]\,\mathbb{E}[u'(Y_{t+1})]}\right)$$

Where we would expect $tp_t > 0$ if $cov(\cdot) < 0$

General Case

For an n period bond, we get:

$$i_{t,t+n} \approx \frac{1}{n}\left[i_{t,t+1} + i^e_{t+1,t+2} + \ldots i^e_{t+n-1,t+n}\right] + \frac{n-1}{n} tp_t$$

Where tp_t is exactly the same as before – based on covariance between future one period bond price and future marginal utility

So as n gets bigger, $\frac{n-1}{n} tp_t$ gets bigger (at a decreasing rate)

This delivers positive slope of typical yield curve, but expectations hypothesis logic drives movements in short and long term yields

A First Look at Quantitative Easing and Unconventional Monetary Policy

The interest rates relevant for most investment and consumption decisions are long-term (e.g., mortgage) and risky (e.g. Baa corporate bond)

Conventional monetary policy targets short-term and riskless interest rates (e.g. Fed Funds Rate)

In practice, something like a 10 Year Treasury yield serves as a benchmark interest rate for all other kinds of interest rates – mortgage rates, credit card rates, student loan rates

These are the rates relevant for economic decisions on spending and saving

Our theory of bond pricing helps us understand the connection between the these different kinds of interest rates

Monetary Policy in Normal Times

Think of a world where the liquidity premium theory holds

A central bank lowers (raises) short term interest rates and is expected to keep these low (high) for some time

This ought to also lower (raise) longer term rates to the extent to which longer term rates are the average of expected shorter term rates

Holding risk factors constant, substitutability between bonds means that riskier yields also ought to fall (increase)

Monetary Policy Simulation

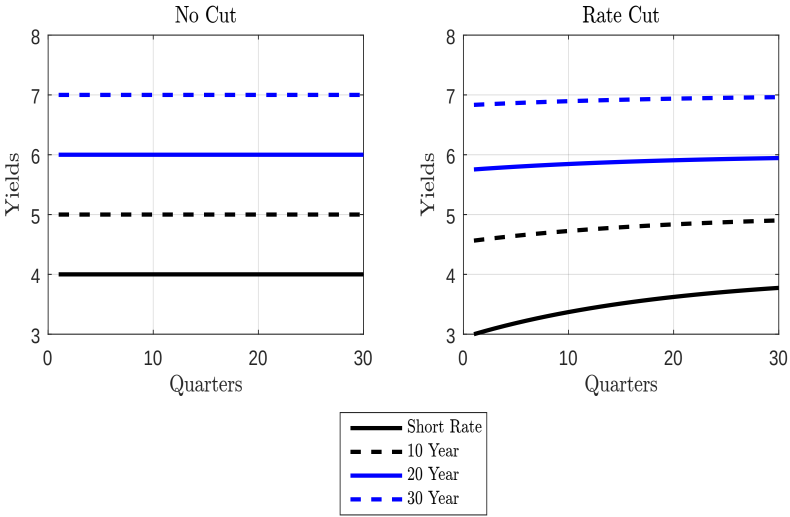
Consider 1 year, 10 year, 20 year, and 30 year bonds

Fixed liquidity premia of 1, 2, and 3

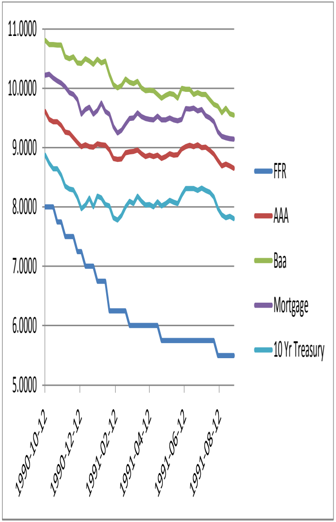
Short-term rate is cut from 4 to 3 persistently

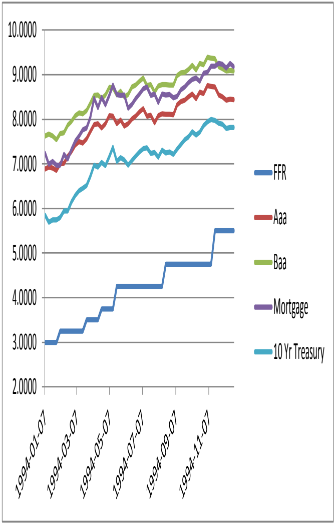
Simulate yields for 30 quarters

Theoretical Simulation

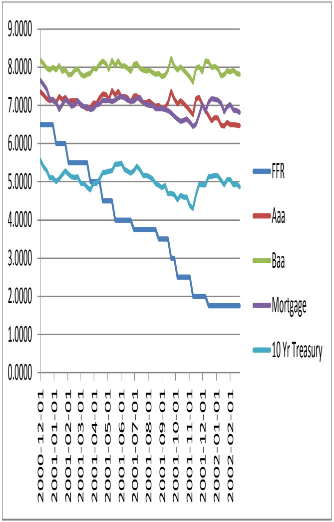


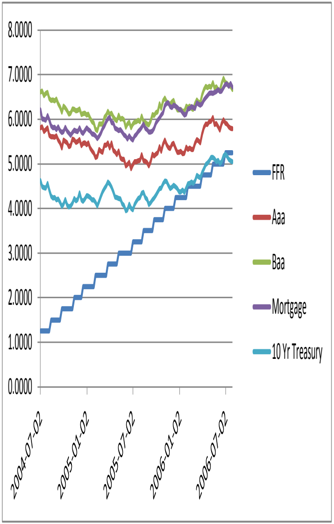
Monetary Loosening and Tightening in Historical Episodes





Monetary Loosening and Tightening in Historical Episodes





Term Structure Puzzle?

If you look at last two pictures, in the 1990s other rates tracked the FFR reasonably closely, but this falls apart in the 2000s

This has been referred to as the term structure puzzle and is discussed in Poole (2005)

From perspective of expectations hypothesis, would expect longer maturity, riskier rates to move along with shorter maturity rates. Not what we see in 2000s

But two key issues:

- Forecastability: if people expected the Fed to raise rates starting in 2004/2005 from the perspective of 2002/2003, this would have been incorporated into long rates before the FFR started to move. Transparency (Crowe and Meade 2007)
- Persistence: behavior of long rates depends on how persistent changes in short rates are expected to

Unconventional Monetary Policy

In a world where Fed Funds Rate is at or very near zero, conventional monetary loosening isn't on the table (ZLB)

Unconventional monetary policy:

1. Quantitative Easing (or Large Scale Asset Purchases): purchases of longer maturity government debt or risky private sector debt. Idea: raise demand for this debt, raise price, and lower yield. See here
2. Forward Guidance: promises to keep future short-term interest rates low. Idea is to work through expectations hypothesis and to lower long term yields immediately. See here

How Might QE Work?

Ben Bernanke: "The problem with quantitative easing is that it works in practice but not in theory"

- Under expectations hypothesis, <u>cannot</u> affect long-term yields without impacting path of short-term yields
- Not clear why Fed ought to be able to impact liquidity/term premium, which depends on a covariance term
- Some form of segmented markets?