# Money Demand, Prices, Inflation, and Interest Rates

## ECON 40364: Monetary Theory & Policy

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# Readings

Mishkin Ch. 19

Friedman, Ch. 2 (section “The Demand for Money” through the end of the chapter)

# Classical Monetary Theory

We have now defined what money is and how the supply of money is set

What determines the demand for money?

How do the demand and supply of money determine the price level, interest rates, and inflation?

We will focus on a framework in which money is neutral and the classical dichotomy holds: real variables (such as output and the real interest rate) are determined independently of nominal variables like money

We can think of such a world as characterizing the "medium" or "long" runs (periods of time measured in several years)

# Velocity

Let $Y_t$ denote real output in period $t$ (units = goods)

$P_t$ is the dollar price of output, so $P_t Y_t$ is the dollar value of output (i.e., nominal GDP)

$\frac{1}{P_t}$ is the "price" of money measured in terms of goods

Define velocity as the average number of times per year that the typical unit of money, $M_t$, is spent on goods and serves. Denote by $V_t$

# Equation of Exchange

The "equation of exchange" or "quantity equation" is:

$$M_t V_t = P_t Y_t$$

This equation is an identity and defines velocity as the ratio of nominal GDP to the money supply:

$$V_t = \frac{P_t Y_t}{M_t}$$

# From Equation of Exchange to Quantity Theory

The quantity equation can be interpreted as a theory of money demand by making assumptions about velocity:

$$M_t = \frac{1}{V_t} P_t Y_t$$

Monetarists: velocity is determined primarily by payments technology (e.g., credit cards, ATMs, etc) and is therefore close to constant (or at least changes are low frequency and therefore predictable)

Let $\kappa = V_t^{-1}$ and treat it as constant. Since money demand, $M_t^d$, equals money supply, $M_t$, our money demand function is:

$$M_t^d = \kappa P_t Y_t$$

Money demand proportional to nominal income; $\kappa$ does not depend on things like interest rates

This is called the quantity theory of money

# Velocity, Money Demand, and the Quantity Theory

The terms "velocity" and "money demand" are often used interchangeably

Re-write in terms of real balances (purchasing power of money):

$$\frac{M_t}{P_t} = \frac{1}{V_t} Y_t$$

The demand for real balance is proportional to the real quantity of exchange

$\frac{1}{V_t}$ is the demand "shifter" – demand for money goes up, means velocity goes down

Quantity theory of money: <u>assumes</u> velocity is roughly constant (equivalently, demand for money is stable)

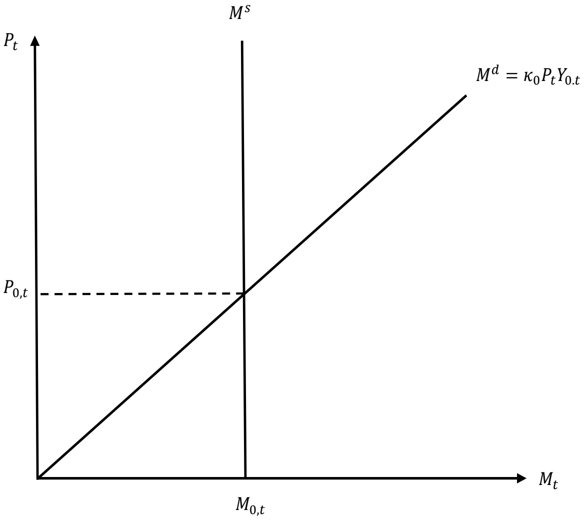
# Demand-Supply Interpretation

Assume money supply is exogenously "set" by the central bank

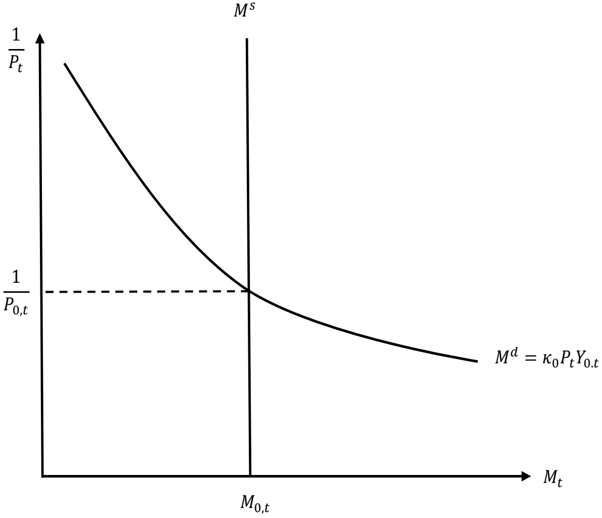
Graph demand for money as *upward-sloping* in $P_t$ (taking $Y_t$ and $\kappa = \frac{1}{V_t}$ as given). Alternatively, *downward-sloping* in $1/P_t$

1. Increase in money supply: $P_t$ rises
2. Increase in money demand (decrease in velocity): $P_t$ falls
3. Increase in $Y_t$: $P_t$ falls

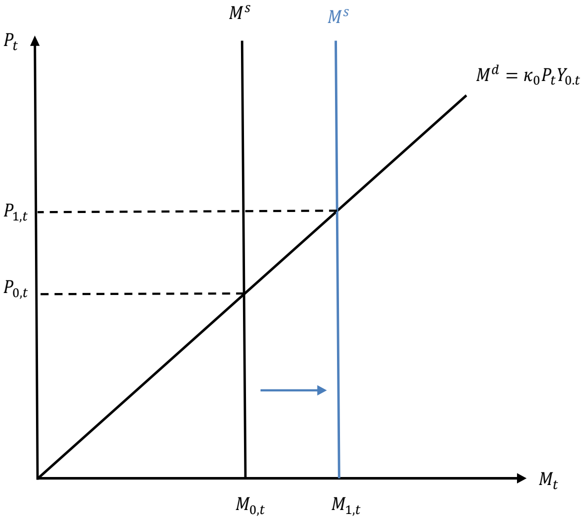
# Demand-Supply Graph



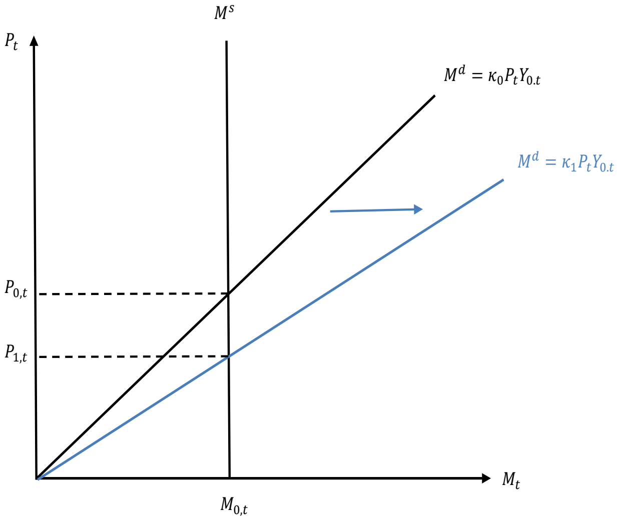
# Alternative Demand-Supply Graph



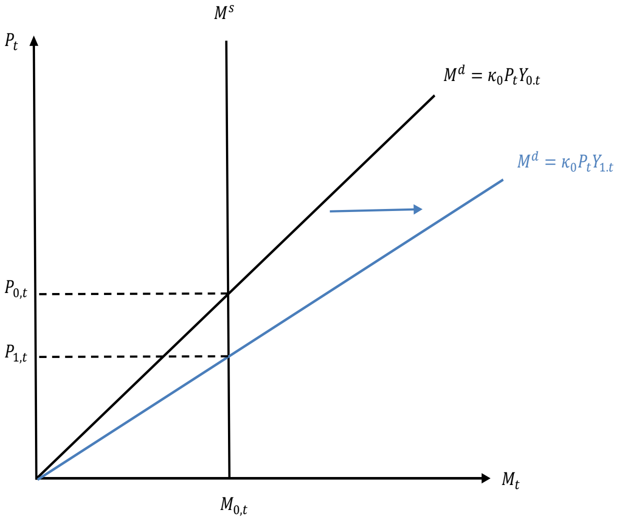
# Increase in Money Supply



# Increase in Money Demand (Decrease in Velocity)



# Increase in $Y_t$ (Increase in Money Demand)



# Money and Prices

Take natural logs of equation of exchange:

$$\ln M_t + \ln V_t = \ln P_t + \ln Y_t$$

If $V_t$ is constant and $Y_t$ is exogenous with respect to $M_t$, then:

$$d \ln M_t = d \ln P_t$$

In other words, a change in the money supply results in a proportional change in the price level (i.e., if the money supply increases by 5 percent, the price level increases by 5 percent)

# Money and Inflation

Since the quantity equation holds in all periods, we can first difference it across time:

$$(\ln M_t - \ln M_{t-1}) + (\ln V_t - \ln V_{t-1}) = (\ln P_t - \ln P_{t-1}) + (\ln Y_t - \ln Y_{t-1})$$

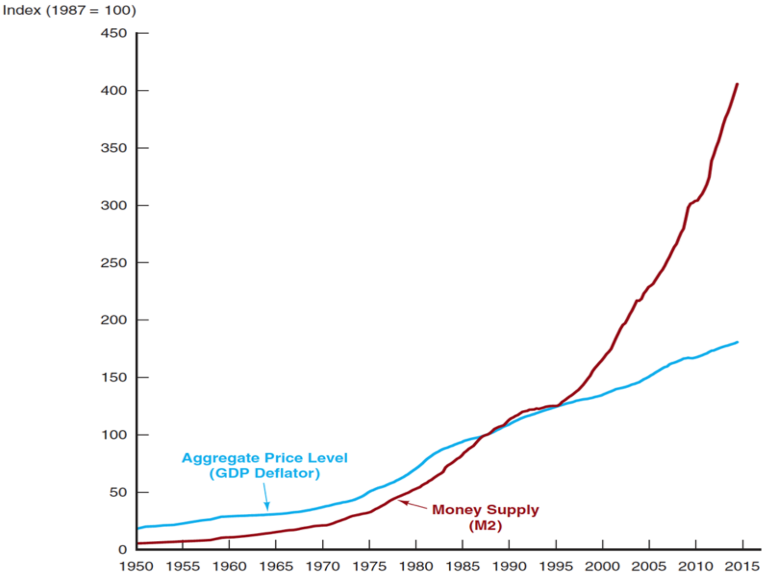
The first difference of logs across time is approximately the growth rate

Inflation, $\pi_t$, is the growth rate of the price level. Constant velocity implies:

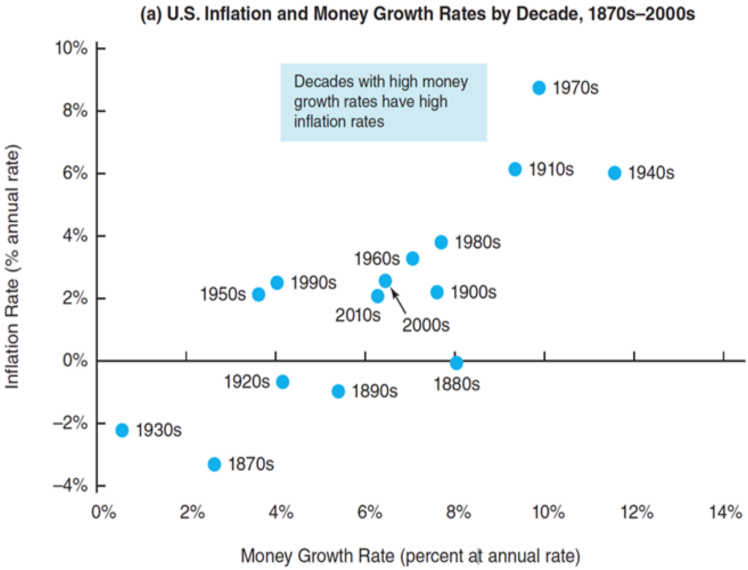
$$\pi_t = g_t^M - g_t^Y$$

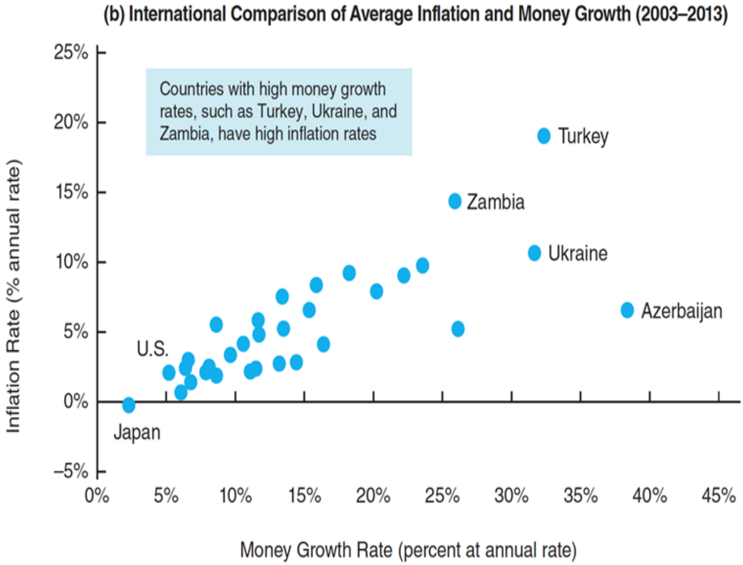
Inflation is the difference between the growth rate of money and the growth rate of output <u>if velocity (money demand) is constant</u>

If output growth were constant, then inflation and money growth would be perfectly correlated



Index (1987 = 100)
450
400
350
300
250
200
150
100
50
0
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Aggregate Price Level
(GDP Deflator)
Money Supply
(M2)


(a) U.S. Inflation and Money Growth Rates by Decade, 1870s–2000s
Decades with high money growth rates have high inflation rates
Inflation Rate (% annual rate)
10%
8%
6%
4%
2%
0%
–2%
–4%
1970s
1910s
1940s
1980s
1960s
1990s
1950s
1900s
2010s
2000s
1920s
1890s
1880s
1930s
1870s
0%
2%
4%
6%
8%
10%
12%
14%
Money Growth Rate (percent at annual rate)


(b) International Comparison of Average Inflation and Money Growth (2003–2013)
Countries with high money growth rates, such as Turkey, Ukraine, and Zambia, have high inflation rates
Turkey
Zambia
Ukraine
Azerbaijan
U.S.
Japan
25%
20%
15%
10%
5%
0%
–5%
0%
5%
10%
15%
20%
25%
30%
35%
40%
45%
Inflation Rate (% annual rate)
Money Growth Rate (percent at annual rate)

# Nominal and Real Interest Rates

The nominal interest rate tells you what percentage of your nominal principal you get back (or have to pay back, in the case of borrowing) in exchange for saving your money. Denote by $i_t$

There are many interest rates, differing by time to maturity and risk. Ignore this for now. Think about one-period (riskless) interest rates – i.e. between $t$ and $t+1$

The real interest rate tells you what percentage of a good you get back (or have to pay back, in the case of borrowing) in exchange for saving. Denote by $r_t$

Putting one good "in the bank" $\Rightarrow$ $P_t$ dollars in bank $\Rightarrow$ $(1+i_t)P_t$ dollars tomorrow $\Rightarrow$ purchases $(1+i_t)\frac{P_t}{P_{t+1}}$ goods tomorrow

# The Fisher Relationship

The relationship between the real and nominal interest rate is then:

$$1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}$$

Since the inverse of the ratio of prices across time is the expected gross inflation rate, we have:

$$1 + r_t = \frac{1 + i_t}{1 + \pi^e_{t+1}}$$

Here $\pi^e_{t+1}$ is expected inflation between $t$ and $t + 1$.
Approximately:

$$r_t = i_t - \pi^e_{t+1}$$

# Classical Dichotomy

In the classical dichotomy, $r_t$ is independent of anything nominal

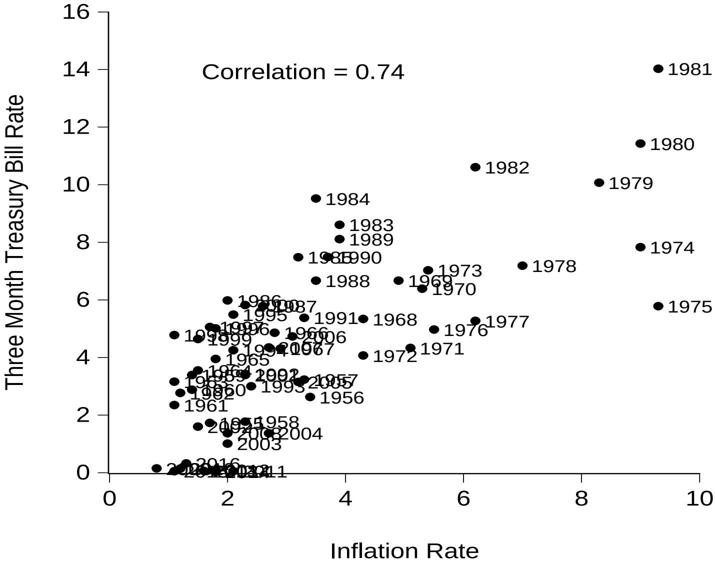
So $i_t$ moves one-for-one with $\pi^e_{t+1}$, holding $r_t$ fixed:

$$i_t = r_t + \pi^e_{t+1}$$

What drives $\pi^e_{t+1}$? Plausible that it's realized inflation (adaptive expectations):

$$i_t = r_t + \pi_t$$

So, there might be a tight connection between inflation and nominal interest rates (to extent to which $r_t$ doesn't move around a ton)


Correlation = 0.74
Three Month Treasury Bill Rate
Inflation Rate
1981
1980
1982
1979
1984
1983
1989
1974
1978
1973
1988
1970
1975
1968
1977
1976
1971
1972
1956
2003
0
2
4
6
8
10
12
14
16

# Theoretical Predictions

The basic quantity theory in which the classical dichotomy holds (real output, real output growth, and the real interest rate independent of nominal things) makes a number of stark predictions:

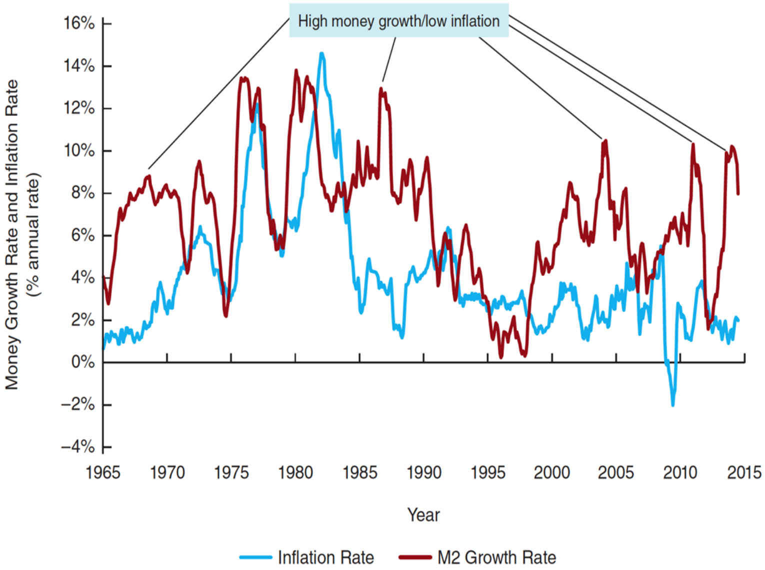
1. The level of the money supply and the price level are closely linked
2. The growth rate of the money supply and the inflation rate are closely linked
3. The inflation rate and the nominal interest rate are closely linked
    - This is not an implication of quantity theory per se – follows from Fisher relationship plus classical dichotomy / monetary neutrality

# Problems with the Quantity Theory

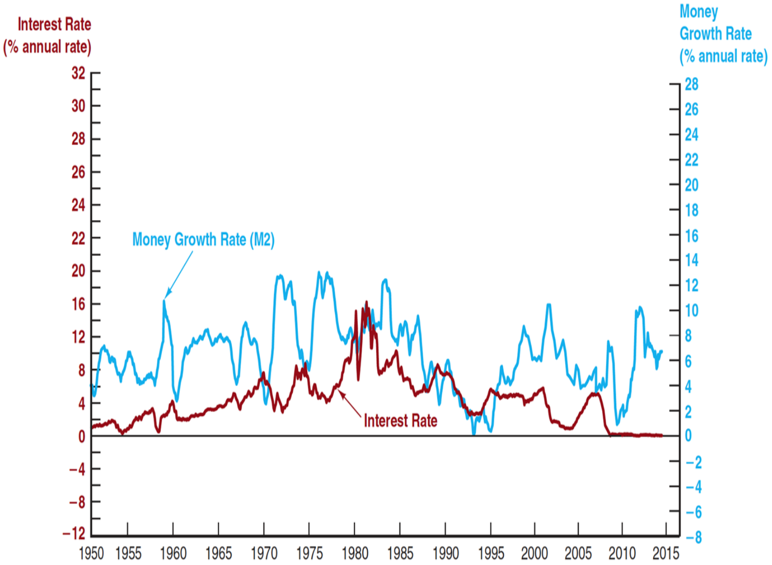
The quantity theory seems to provide a pretty good theory of inflation and interest rates over long horizons as well as in a cross section of countries

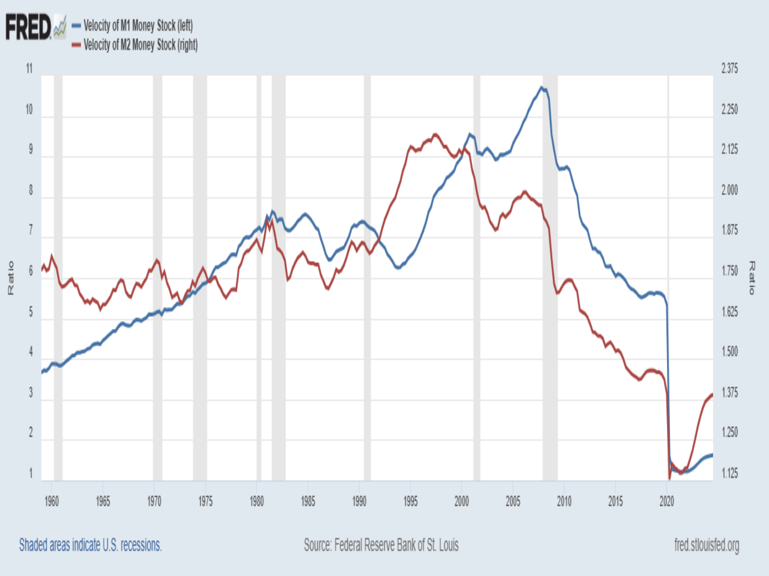
What about the short run?

- The shorter-term relationships between money growth and both inflation and nominal interest rates are weak
- Velocity is not constant and has become harder to predict, particularly since the early 1980s



High money growth/low inflation
Money Growth Rate and Inflation Rate
(% annual rate)
16%
14%
12%
10%
8%
6%
4%
2%
0%
−2%
−4%
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Year
Inflation Rate
M2 Growth Rate


Interest Rate
(% annual rate)
Money
Growth Rate
(% annual rate)
Money Growth Rate (M2)
Interest Rate
32
30
28
26
24
22
20
16
12
8
4
0
−4
−8
−12
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
−2
−4
−6
−8
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015


FRED
Velocity of M1 Money Stock (left)
Velocity of M2 Money Stock (right)
Ratio
11
10
9
8
7
6
5
4
3
2
1
2.375
2.250
2.125
2.000
1.875
1.750
1.625
1.500
1.375
1.250
1.125
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Shaded areas indicate U.S. recessions.
Source: Federal Reserve Bank of St. Louis
fred.stlouisfed.org

# Moving Beyond the Quantity Theory

The key assumption in the quantity theory is that the demand for money (i.e. velocity) is stable (or at least predictable) – you hold money to buy stuff, and how much money you need is proportional to how much you buy

Liquidity preference theory of money demand: money competes with other assets as a store of value. Money is more liquid (can be used in exchange), but how much you want to hold depends on return on other assets

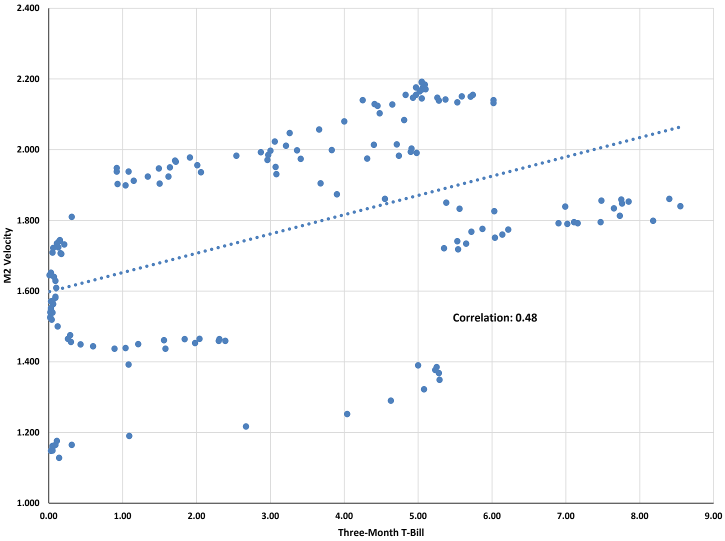
# Liquidity Preference

Demand for real money balances, $m_t = \frac{M_t}{P_t}$, is an increasing function of output, $Y_t$, but a decreasing function of the nominal interest rate, $i_t$:

$$\frac{M_t}{P_t} = L(\underset{-}{i_t}, \underset{+}{Y_t})$$

But then velocity:

$$V_t = \frac{P_t Y_t}{M_t} = \frac{Y_t}{L(i_t, Y_t)}$$


2.400
2.200
2.000
1.800
1.600
1.400
1.200
1.000
M2 Velocity
Correlation: 0.48
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
Three-Month T-Bill

# Two Simple Models

We can generate a liquidity preference theory of money demand via two different setups:

1. Baumol-Tobin: this is an intratemporal portfolio allocation problem. Given desired spending, how to allocate wealth between money and bonds (which pay interest)

2. Money in the Utility Function (MIU): this is an intertemporal problem with both a consumption-saving decision and a portfolio allocation problem (left as exercise)

Both generate something like: $m_t = L(i_t, Y_t)$

# Baumol-Tobin

You need to spend $Y$ over the course of a period (say, a year). This is given.

You have sufficient wealth to do this

Average holdings of illiquid wealth (over the period) earn nominal return $i$

Need to determine how much money (liquid wealth) to hold to hold, which earns nothing. Have to support transactions with money

You can liquidate illiquid wealth (i.e., withdraw money) as often as you please, but each liquidation incurs a "shoeleather cost" of $K \geq 0$

# One “Trip to the Bank”

Suppose you withdraw all the funds you need at the beginning of a period. So you make one trip

Then your average real balance holdings over the period are $Y/2$

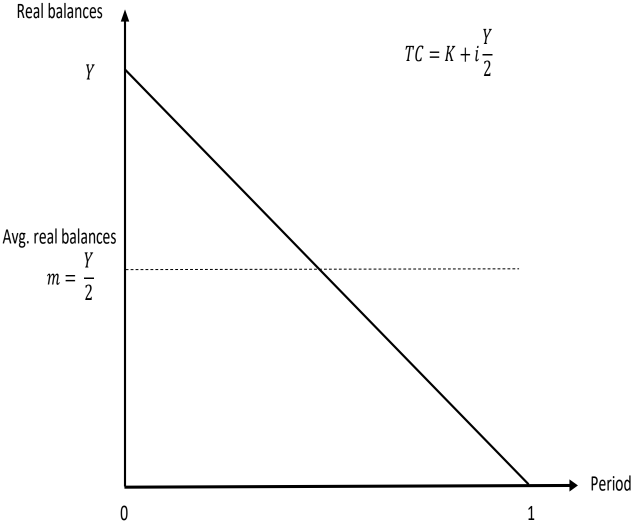
You forego $iY/2$ in interest by holding money instead of bonds

And pay a shoeleather cost of $K$

Total cost is:

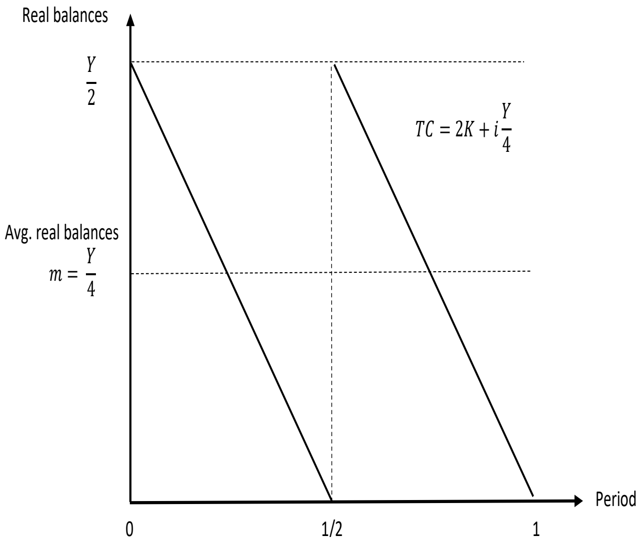
$$TC = K + \frac{iY}{2}$$

# One Trip



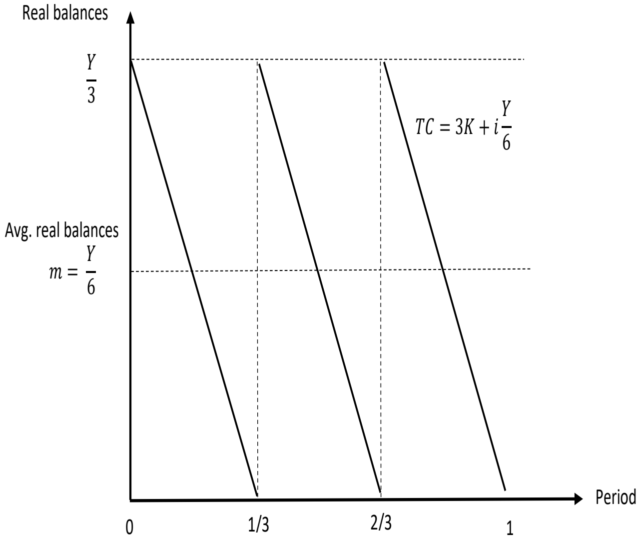
Real balances
$TC = K + i\frac{Y}{2}$
$Y$
Avg. real balances
$m = \frac{Y}{2}$
0
1
Period

# Two Trips



Real balances
$\frac{Y}{2}$
$TC = 2K + i\frac{Y}{4}$
Avg. real balances
$m = \frac{Y}{4}$
Period
0
1/2
1

# Three Trips



Real balances
$\frac{Y}{3}$
$TC = 3K + i\frac{Y}{6}$
Avg. real balances
$m = \frac{Y}{6}$
Period
0
1/3
2/3
1

# General Case

Total cost as a function of trips, $T$, is:

$$TC = TK + i\frac{Y}{2T}$$

Average real balance holdings:

$$m = \frac{Y}{2T}$$

Re-write total cost in terms of $m$ instead of $T$:

$$TC = \frac{KY}{2m} + im$$

# Money Demand Function

Use calculus to get first order condition:

$$m = \sqrt{\frac{KY}{2i}}$$

Or re-arranging:

$$m = \left(\frac{KY}{2}\right)^{\frac{1}{2}} i^{-\frac{1}{2}}$$

Demand for real balances increasing in $Y$ and decreasing in $i$

# Money in the Utility Function

Suppose that there is a representative household who receives utility from consuming goods and holding real money balances, $m_t = \frac{M_t}{P_t}$. Flow utility:

$$U\left(C_t, \frac{M_t}{P_t}\right) = \ln C_t + \psi \ln\left(\frac{M_t}{P_t}\right)$$

Flow budget constraint:

$$P_t C_t + B_t - B_{t-1} + M_t - M_{t-1} \leq P_t Y_t - P_t T_t + i_{t-1} B_{t-1}$$

$B_{t-1}$ and $M_{t-1}$: stocks of bonds and money household enters $t$ with

# Money in the Utility Function Continued

This is an intertemporal problem: household is choosing how much to save in bonds vs. money

Money pays no interest, but provides utility benefit (makes conducting transactions) easier

Generates qualitatively the same money demand function – demand for money is decreasing in interest rate and increases in volume of transactions

Zero lower bound (ZLB): if $i_t \to 0$, there is no reason to hold bonds. Arbitrage forces $i_t \geq 0$.

# Friedman Rule

Milton Friedman argued that optimal monetary policy in the medium to long run would target a nominal interest rate of zero

With a positive real rate of interest, this would require deflation

# Friedman Rule Intuition

A positive nominal interest rates dissuades people from holding money by increasing the opportunity cost of liquidity relative to bonds

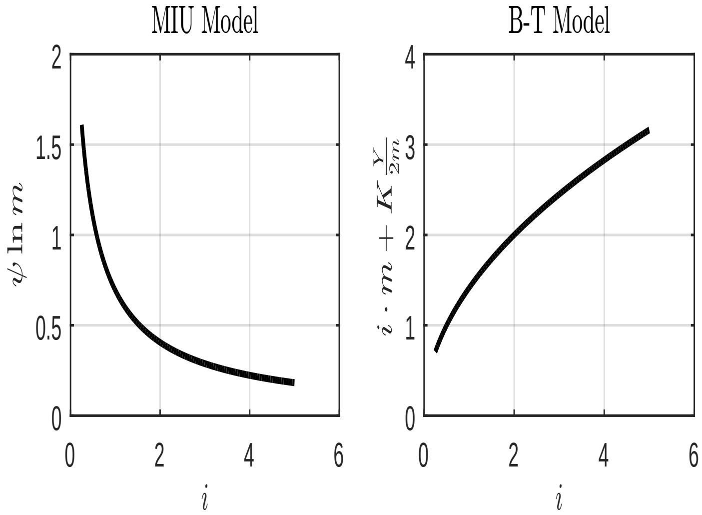
Money is beneficial to support exchange, and the marginal cost of producing (fiat) money is essentially zero

Holds in both the MIU model ($i = 0$ maximizes utility) and the B-T model ($i = 0$ minimizes the cost of holding money)

Why don't central banks follow Friedman rule? Because of the zero lower bound and short run stabilization policy

Helps understand the two-percent inflation target: balances Friedman rule desire to drive nominal rates toward zero with objective to use interest rates for stabilization policy

# Optimality of $i = 0$



# Money and Inflation: The Case of Hyperinflations

Milton Friedman famously said that "inflation is everywhere and always a monetary phenomenon"

Simple logic based on the quantity equation. Works pretty well in the long run

What about extreme situations of inflation, or what are called "hyperinflations"?

Seem to be monetary phenomena triggered by fiscal problems

# Hyperinflations

**Table 8.1** Hyperinflations in History

| Country | Year | Highest Inflation per Month % | Country | Year | Highest Inflation per Month % |
|---|---|---|---|---|---|
| Argentina | 1989/90 | 196 | Hungary | 1945/46 | $1.295*10^{16}$ |
| Armenia | 1993/94 | 438 | Kazakhstan | 1994 | 57 |
| Austria | 1921/22 | 124 | Kyrgyzstan | 1992 | 157 |
| Azerbaijan | 1991/94 | 118 | Nicaragua | 1986/89 | 127 |
| Belarus | 1994 | 53 | Peru | 1921/24 | 114 |
| Bolivia | 1984/86 | 120 | Poland | 1989/90 | 188 |
| Brazil | 1989/93 | 84 | Poland | 1992/94 | 77 |
| Bulgaria | 1997 | 242 | Serbia | 1922/24 | 309,000,000 |
| China | 1947/49 | 4,209 | Soviet Union | 1945/49 | 279 |
| Congo (Zaire) | 1991/94 | 225 | Taiwan | 1995 | 399 |
| France | 1789/96 | 143 | Tajikistan | 1993/96 | 78 |
| Georgia | 1993/94 | 197 | Turkmenistan | 1992/94 | 63 |
| Germany | 1920/23 | 29,500 | Ukraine | 1990 | 249 |
| Greece | 1942/45 | 11,288 | Yugoslavia | | 59 |
| Hungary | 1923/24 | 82 | | | |

SOURCE: Peter Bernholz, *Monetary Regimes and Inflation: History, Economic and Political Relationships* (Edward Elgar Publishing, March 27, 2006).

# Hyperinflations Usually a Fiscal Phenomenon

Most hyperinflations in history are associated with fiscal mischief

Government's budget constraint:

$$P_t G_t + i_{t-1} B_{G,t-1} = P_t T_t + P_t T_{cb,t} + B_{G,t} - B_{G,t-1}$$

Here $P_t$ is the nominal price of goods (i.e., the price level), $B_{G,t-1}$ is the stock of debt with which a government enters period $t$, $B_{G,t}$ is the stock of debt the government takes from $t$ to $t+1$, $i_{t-1}$ is the nominal interest rate on that debt, $T_t$ is tax revenue (real), and $T_{cb,t}$ is a transfer from central bank

# Consolidated Budget Constraint

Central bank's budget constraint:

$$B_{cb,t} - B_{cb,t-1} + P_t T_{cb,t} = M_t - M_{t-1} + i_{t-1} B_{cb,t-1}$$

Consolidated government budget constraint (combine the two, with market-clearing condition that $B_{G,t} = B_{cb,t} + B_t$, where $B_t$ is bonds held by public):

$$P_t G_t + i_{t-1} B_{t-1} = P_t T_t + M_t - M_{t-1} + B_t - B_{t-1}$$

# Monetizing the Debt

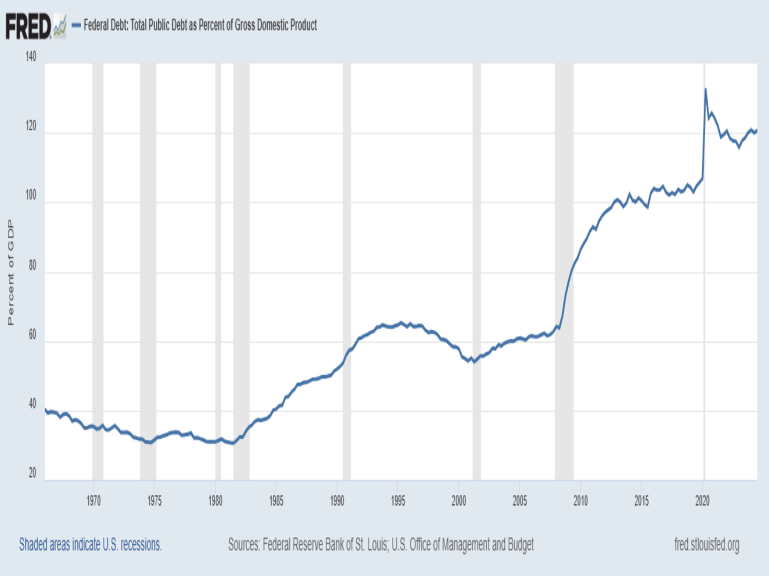
Fiscal deficit equals change in money supply plus change in debt:

$$P_t G_t + i_{t-1} B_{t-1} - P_t T_t = M_t - M_{t-1} + B_t - B_{t-1}$$

If tax revenue doesn't cover expenditure (spending plus interest on debt), then government either has to issue more debt or "print more money"

Monetizing the debt: fiscal authority issues debt to finance deficit, but monetary authority buys the debt by doing open market operations, which creates base money, and the debt effectively doesn't end up in hands of the public (i.e. $B_{G,t}$ goes up, but this is absorbed by $B_{cb,t}$, so it doesn't appear as $B_t$)

# Time to Worry?



# Application: Seigniorage and the Inflation Tax

Nominal revenue from printing money is: $M_t - M_{t-1}$

Real revenue from printing money is $\frac{M_t - M_{t-1}}{P_t}$

We call the real revenue from printing money seigniorage

$$\text{Seigniorage} = \frac{M_t - M_{t-1}}{P_t}$$

This can equivalently be written:

$$\text{Seigniorage} = \frac{M_t - M_{t-1}}{M_{t-1}} \frac{M_{t-1}}{M_t} \frac{M_t}{P_t}$$

# Simplification

Define the growth rate of money as:

$$g_t^M = \frac{M_t - M_{t-1}}{M_{t-1}}$$

Then the expression for seigniorage can be written:

$$\text{Seigniorage} = \frac{g_t^M}{1 + g_t^M} m_t$$

Or approximately:

$$\text{Seigniorage} = g_t^M m_t$$

$g_t^M$ is effectively the "tax rate" and $m_t$ is the "tax base"

# Long Run: Some Assumptions

Suppose that the real interest rate is constant and invariant to nominal variables (classical dichotomy)

Fisher relationship:

$$i = r + \pi$$

Suppose that the inflation rate equals the money growth rate (output and nominal rate constant)

$$i = r + g^M$$

Demand for real balances:

$$m = L(r + g^M, Y)$$

# "Optimal" Inflation Tax

Suppose that a central bank wants to pick $g^M$ to maximize seigniorage

$$\max_{g^M} \quad g^M L(r + g^M, Y)$$

Provided money demand is decreasing in nominal interest rate (i.e. $L_i(\cdot) < 0$), then two competing effects of higher $g^M$:

1. Tax rate: higher $g^M \Rightarrow$ higher tax rate
2. Base: higher $g^M \Rightarrow$ lower tax base

# First-Order Condition

$$g^M = -\frac{L(r + g^M, Y)}{L_i(r + g^M, Y)}$$

Revenue-maximizing growth rate of money inversely related to interest sensitivity of money demand

If money demand interest insensitive (e.g., quantity theory), then revenue-maximizing $g^M = \infty$!

Desire for seigniorage another reason to move away from Friedman rule