Problem Set 1

ECON 30020, Intermediate Macroeconomics, Spring 2026
The University of Notre Dame
Professor Sims

Instructions: Please prepare legible, complete solutions to the following problems. You may work with others, and consult whatever resources you wish, but you are responsible for your own work and must turn in your own assignment. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers and/or figures from Excel – you need not turn in Excel code. An image/scan of the problem set should be uploaded onto the Canvas site via the "assignments" tab no later than 5:00 pm on the due date of January 29.

1. **The Rule of 70** Suppose that you have a variable, X_t, growing at a constant rate of g. Relative to period t, in period $t+h$ that variable will therefore be:

$$X_{t+h} = (1+g)^h X_t$$

   (a) Create an Excel file. Create a column labeled "Period" and have periods run from 0 (the first period) to 100 (the last period). Normalize the period 0 value, X_0, to 1. Consider different growth rates: 2, 5, 7, 10, and 20 percent. For each growth rate, approximately how many periods does it take for the variable X to double?

   (b) The "Rule of 70" provides a quick approximation to calculate how many periods it will take a variable to double. In particular, if a variable is growing at rate $G = g \times 100$, the Rule of 70 says that variable out to double approximately every $70/G$ periods. According to the Rule of 70, approximately how many periods should it take X to double for growth rates of 2, 5, 7, 10, and 20 percent?

   (c) Compare your answers from (b) (the approximation based on the rule of 70) and (a) (the actual number of periods to double). Comment on the quality of the approximation.

2. **Speed of Convergence:** Consider the textbook Solow model. Normalize the levels of productivity and labor supply to unity (i.e., $A = 1$ and $N_t = 1$). Assume a Cobb-Douglas production function, i.e., $Y_t = K_t^\alpha N_t^{1-\alpha}$. In per-worker terms, the central equation is:

$$k_{t+1} = s k_t^\alpha + (1-\delta) k_t$$

   (a) Solve for an analytic expression for steady-state capital.

   (b) Create an Excel or Google Sheets file. Suppose $s = 0.2$, $\delta = 0.1$, and $\alpha = 1/3$. Numerically solve for the steady-state capital stock.

   (c) In your Excel file, create a column of periods (running from 0 to 100). In period 0, suppose that the initial capital stock is 1/2 its steady-state value. Trace out the series of capital per worker across time. Print this out in your solution. Confirm that the capital stock per worker converges to the steady state.

(d) Calculate the approximate number of periods for the capital stock per worker to get half way to its steady state (i.e., calculate the "half-life").

(e) Re-do this exercise, but with two different values of α: $\alpha = 3/20$ and $\alpha = 1/2$. Calculate the half-lives. What is the relationship between the value of α and the half-life of the capital stock per worker?

3. GLS, Chapter 5, Exercise 2.

4. **Optimal Saving Rate in the Solow Model:** Consider a standard Solow model, with Cobb-Douglas production and productivity normalized to $A = 1$. Also normalize $N_t = 1$. The central equation of the model, written in per-worker terms, is:

$$k_{t+1} = sk_t^{\alpha} + (1 - \delta)k_t$$

(a) Derive expressions for the steady-state capital stock and consumption per worker.

(b) Derive an expression for the saving rate that maximizes consumption per worker in the steady state (i.e., derive the Golden Rule saving rate).

5. **An Increase in the Rate of Labor-Augmenting Productivity:** Consider a Solow model with population and labor-augmenting productivity growth. The production function is Cobb-Douglas. The central equation is:

$$K_{t+1} = sAK_t^{\alpha} (Z_t N_t)^{1-\alpha} + (1 - \delta)K_t$$

Z_t and N_t grow deterministically according to:

$$\frac{Z_{t+1}}{Z_t} = 1 + z$$

$$\frac{N_{t+1}}{N_t} = 1 + n$$

Where z and n are net growth rates, both greater than or equal to zero.

(a) Re-write the central equation in terms of capital per efficiency unit of labor, $\widehat{k}_t = K_t/(Z_t N_t)$.

(b) Plot the central equation, and argue that there exists a steady state in per efficiency units.

(c) Analytically solve for the steady state capital stock per efficiency unit of labor.

(d) Assume that the economy initially sits in a steady state. Suppose that there is a one-time, permanent increase in z. Graphically, show how this impacts the steady-state capital stock per efficiency unit of labor. Generate a qualitative impulse response graph of how the capital stock per efficiency unit of labor evolves over time.

(e) Now create an Excel file. Assume initially that $s = 0.2$, $\delta = 0.1$, $z = 0.02$, $n = 0.01$, and $A = 1$. Numerically solve for the initial steady-state capital stock per efficiency unit of labor. Create a grid of periods, running from period 0 to period 100. Assume that the values of $Z_0 = N_0 = 1$ (i.e., the values of Z_t and N_t when $t = 0$, the first period, are both

1). Assuming the capital stock per efficiency unit of labor initially sits in this steady state, solve for the dynamic path of capital per worker ($k_t = K_t/N_t$) from periods 0 to 100. Create a plot of the natural log of capital per worker over time.

(f) Now, suppose that in period 10, we have z increase from 0.02 to 0.03 (i.e., the same experiment you did qualitatively above). In Excel, trace out the dynamic path of capital per efficiency unit of labor and capital per worker in response. Create a plot of the natural log of capital per worker over time.

(g) Let the real wage equal the marginal product of labor, i.e., $w_t = \frac{\partial Y_t}{\partial N_t}$. Write down the analytical expression for the real wage with the Cobb-Douglas production function and trend growth in labor augmenting productivity and population. In your Excel file, create a column for the real wage. Trace out the dynamic path of the real wage in response to the one-time increase in z in period 10. Produce a plot of the natural log of the real wage over time.