Problem Set 2

ECON 30020, Intermediate Macroeconomics, Spring 2026
The University of Notre Dame
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Instructions: Please prepare legible, complete solutions to the following problems. You may work with others, and consult whatever resources you wish, but you are responsible for your own work and must turn in your own assignment. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers and/or figures from Excel – you need not turn in Excel code. An image/scan of the problem set should be uploaded onto the Canvas site via the "assignments" tab no later than 5:00 pm on the due date of February 17.

1. **Consumption Function with Perfect Complement Preferences:** Suppose that a household faces the following consumption-saving problem:

$$\max_{C_t, S_t} \quad U = \min(C_t, C_{t+1})$$

s.t.

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1 + r_t)S_t$$

Lifetime utility here is different than our normal setup, which would have $U = u(C_t) + \beta u(C_{t+1})$, where $u(\cdot)$ is some function.

   (a) Solve for the consumption function for C_t as a function of things the household takes as given. HINT: you cannot use calculus here. You have to think.

   (b) Derive an expression for partial derivative of C_t with respect to r_t under three different scenarios:

      i. Income is equal across periods $Y_t = Y_{t+1} = Y > 0$.

      ii. The household only has income in the first period ($Y_t = Y > 0$, $Y_{t+1} = 0$)

      iii. The household only has income in the second period ($Y_t = 0$, $Y_{t+1} = Y > 0$)

   Discuss the sign of these partial effects in terms of income and substitution effects.

2. **Consumption Function with Quasi-Linear Preferences:** Suppose we have a two-period consumption-saving model. Instead of assuming the same, concave flow utility function for periods t and $t + 1$, we are instead going to assume that preferences are quasi-linear. There is no uncertainty. In particular, the household solves the following problem:

$$\max_{C_t, C_{t+1}} \quad U = \ln C_t + \beta C_{t+1}$$

s.t.

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

(a) Derive the consumption Euler equation.

(b) Derive the consumption function. What is the marginal propensity to consume? Provide some intuition.

3. **Optimal Consumption-Saving with a Borrowing Constraint:** Suppose a household has standard preferences and faces the sequence of flow budget constraints as follows:

$$\max_{C_t, S_t} \quad U = \ln C_t + \beta \ln C_{t+1}$$

s.t.

$$C_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + (1+r_t)S_t$$

For now, assume there is nothing unusual about the problem.

(a) Derive the consumption function for a typical household.

(b) Suppose you have a household with income stream $Y_t = 2$ and $Y_{t+1} = 1$. Suppose that $\beta = 0.95$ and $r_t = 0.05$. Solve for the numeric values of C_t and S_t.

(c) Repeat the above problem, but instead assume $Y_t = 1$ and $Y_{t+1} = 2$.

(d) Suppose Y_t increases by 0.1 (i.e., to 2.1 for the household in (b) and to 1.1 for the household in (c)). What is the numeric value of the marginal propensity to consume (i.e., the change in consumption divided by the change in income) for both types of households?

(e) Now, suppose that the household is subject to a borrowing constraint. In particular, $S_t \geq 0$ (i.e., the household cannot go into debt). Repeat the analysis from (a) - (d) with a borrowing constraint. Comment upon how the marginal propensities to consume for the two types of household differ (or do not) when there is a borrowing constraint.

4. **Life-Cycle Consumption/Saving:** Suppose that a household lives for $T+1$ periods, where $T > 1$. There is no uncertainty. Its lifetime utility function is:

$$U = u(C_t) + \beta u(C_{t+1}) + \ldots \beta^T u(C_{t+T})$$

The real interest rate is constant across time, $r > 0$, and $\beta(1+r) = 1$. The household begins with no wealth and dies with no wealth (nor any debt). The sequence of flow budget constraints are:

$$C_t + S_t = Y_t$$

$$C_{t+1} + S_{t+1} = Y_{t+1} + (1+r)S_t$$

$$\vdots$$

$$C_{t+T} = Y_{t+T} + (1+r)S_{t+T-1}$$

(a) With $\beta(1+r) = 1$, what will be true about consumption across time? Given this, derive an analytic expression for consumption in each period.

(b) Suppose that $T = 49$. Suppose that from periods t through $t+9$ income is zero (i.e., $Y_t = Y_{t+1} = \ldots Y_{t+9} = 0$. Suppose that income in period $t+10$ is 10 (i.e., $Y_{t+10} = 10$). Then, suppose that income grows 2 percent per period from period 11 through period 40 (i.e., $Y_{t+10+j} = (1.02)^j Y_{t+10}$, for $j = 0, \ldots 30$.). Then suppose that income from periods $t+41$ through $t+49$ is zero (i.e., $Y_{t+j} = 0$ for $j = 41, \ldots 49$). Suppose that $\beta = 0.95$. In Microsoft Excel or Google Sheets, create a column for the income process, consumption each period, flow saving each period, and the stock of savings brought between each period. Create a plot showing (i) income, (ii) consumption, and (iii) the stock of savings across the household's life cycle.

(c) In your own words, comment on the household's saving behavior across its life.

5. **Consumption Under Uncertainty**: Suppose that a household lives for two periods. It receives a flow of income of $Y_t = 2$ in the first period; this is deterministic. In the second period, it receives a flow of income of $Y_{t+1}(1) = 3$ with probability 1/2 (call this state 1) and a flow of income of $Y_{t+1}(2) = 1$ with probability 1/2 (call this state 2). It can save or borrow in the first period at interest rate r_t. The period t budget constraint is:

$$C_t + S_t = Y_t$$

The second-period budget constraint must hold in both states of the world, so we have:

$$C_{t+1}(1) = Y_{t+1}(1) + (1+r_t)S_t$$
$$C_{t+1}(2) = Y_{t+1}(2) + (1+r_t)S_t$$

The household wants to maximize the expected value of lifetime utility:

$$U = u(C_t) + \beta \mathbb{E} u(C_{t+1}) = u(C_t) + \frac{1}{2}\beta u(C_{t+1}(1)) + \frac{1}{2}\beta u(C_{t+1}(2))$$

You may assume that $\beta(1+r_t) = 1$.

(a) Derive the consumption Euler equation.

(b) Explain, in words, why even though $\beta(1+r_t) = 1$, consumption may not be constant in expectation across periods.

(c) Suppose that the flow utility function is the natural log, i.e., $u(C_t) = \ln C_t$. Assume that $\beta = 0.95$ (and, therefore, $1 + r_t = 0.95^{-1}$). Solve for the optimal value of saving, S_t, as well as consumption in the first period, expected consumption in the second period, and consumption in both states of the world in the second period.

(d) Re-do this analysis, but suppose that $Y_{t+1}(1) = 4$ and $Y_{t+1}(2) = 0$. What happens to saving? Explain briefly.