# Lecture 2: Stylized Facts and Basic Solow Model

## ECON 30020: Intermediate Macroeconomics

Prof. Eric Sims

University of Notre Dame

Spring 2026

# Readings

GLS Ch. 4 (facts)

GLS Ch. 5 (Solow Growth Model)

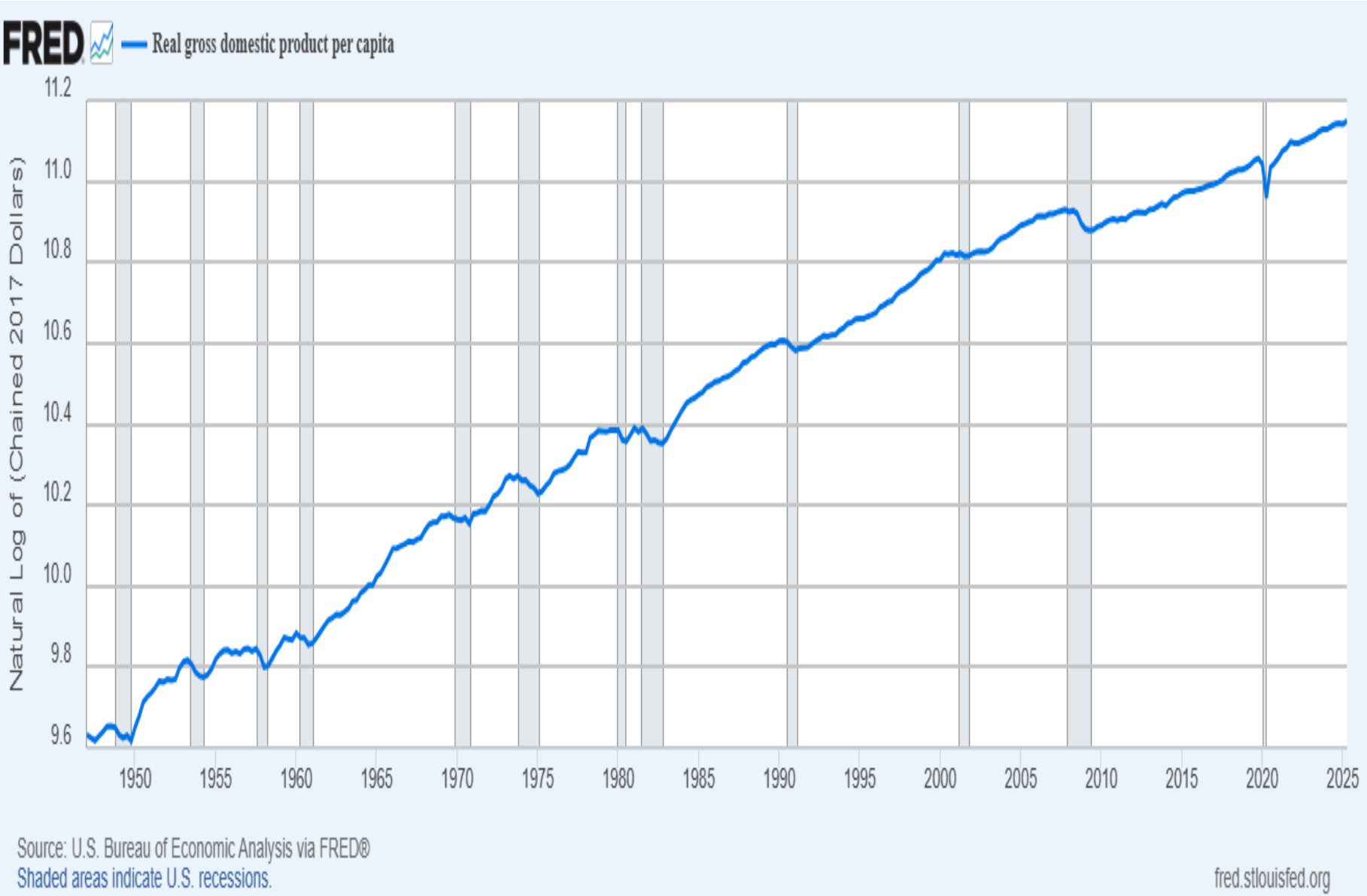
# Economic Growth

When economists say "growth," typically mean average rate of growth in real GDP per capita over long horizons

- Long run: frequencies of time measured in decades
- Not period-to-period fluctuations in the growth rate

"Once one begins to think about growth, it is difficult to think about anything else" – Robert Lucas, 1995 Nobel Prize winner

# US Real GDP per capita



# Summary Stats

Average (annualized) growth rate of per capita real GDP: 1.8%

Implies that the level of GDP doubles roughly once every 40 years

- Growing just 0.2 percentage points faster (2% growth rate): level doubles every 35 years
- Rule of 70: number of years it takes a variable to double is approximately 70 divided by the growth rate
- Consider two countries that start with same GDP, but country $A$ grows 2% per year and country $B$ grows 1% per year. After 100 years, $A$ will be 165% richer!

Small differences in growth rates really matter over long horizons

# Key Question

What accounts for this growth?

In a mechanical sense, can only be two things:

- Growth in productivity: we produce more output given the same inputs
- Factor accumulation: more factors of production help us produce more stuff

# Factors of Production

Two key factors of production on which we focus are capital and labor

- Capital: stuff we produce that we don't consume and instead use to produce other stuff
- Labor: measured in units of time

Hours worked per capita (i.e., labor) is roughly trendless – not a plausible source of growth in per-capita income

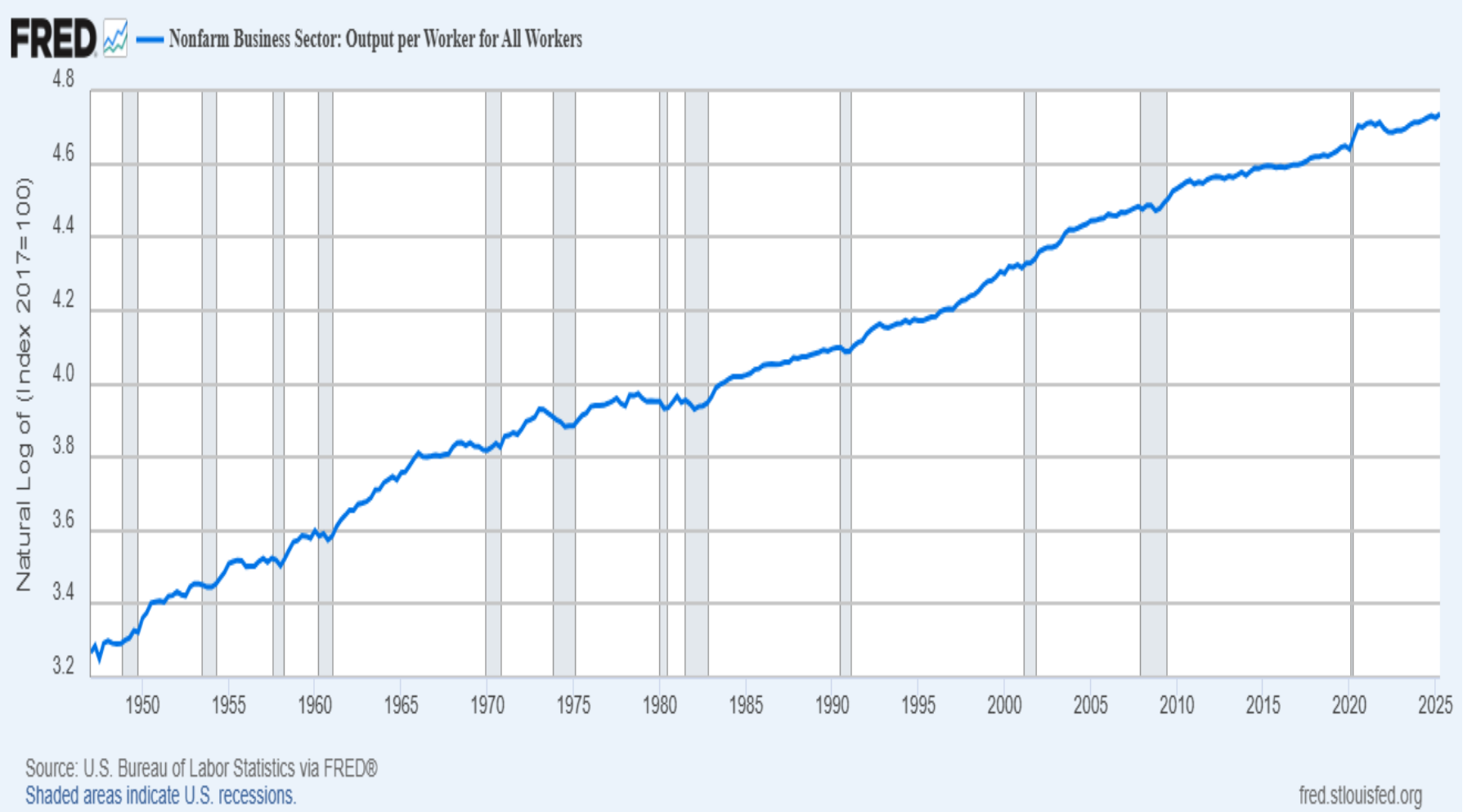
So what drives growth: capital accumulation or productivity improvements?

Relatedly, are rich countries rich because they have more capital than poorer countries, or because they are more productive?

# Stylized Facts: Time Series

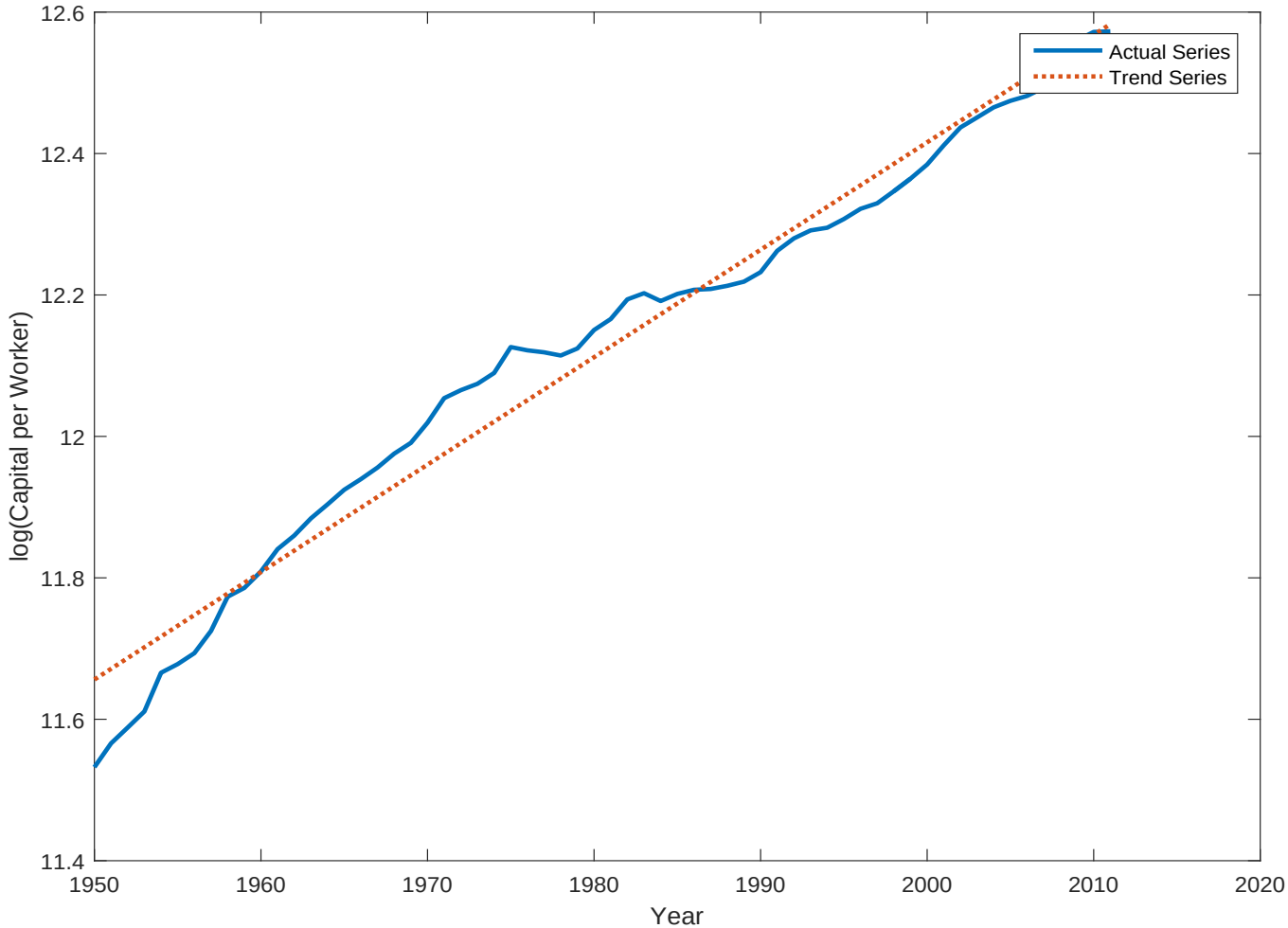
1. Output per worker grows at an approximately constant rate over long periods of time ▸ picture
2. Capital per worker grows at an approximately constant rate over long periods of time ▸ picture
3. The capital to output ratio is roughly constant over long periods of time ▸ picture
4. Labor's share of income is roughly constant over long periods of time (though some decline in recent decades) ▸ picture
5. The return to capital is roughly constant over long periods of time ▸ picture
6. The real wage grows at approximately the same rate as output per worker over long periods of time ▸ picture

# Output Per Worker over Time



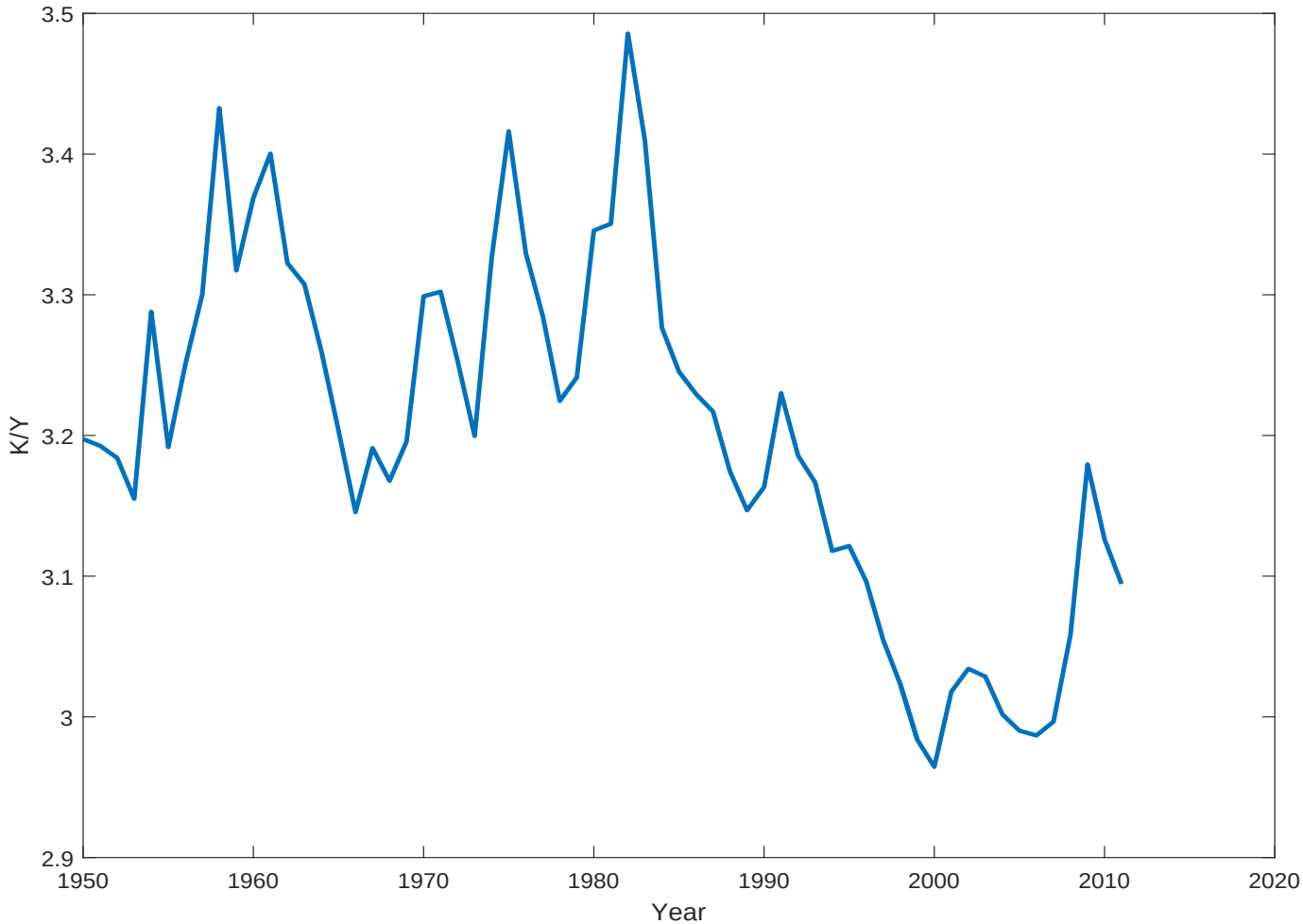

go back

# Capital Per Worker over Time



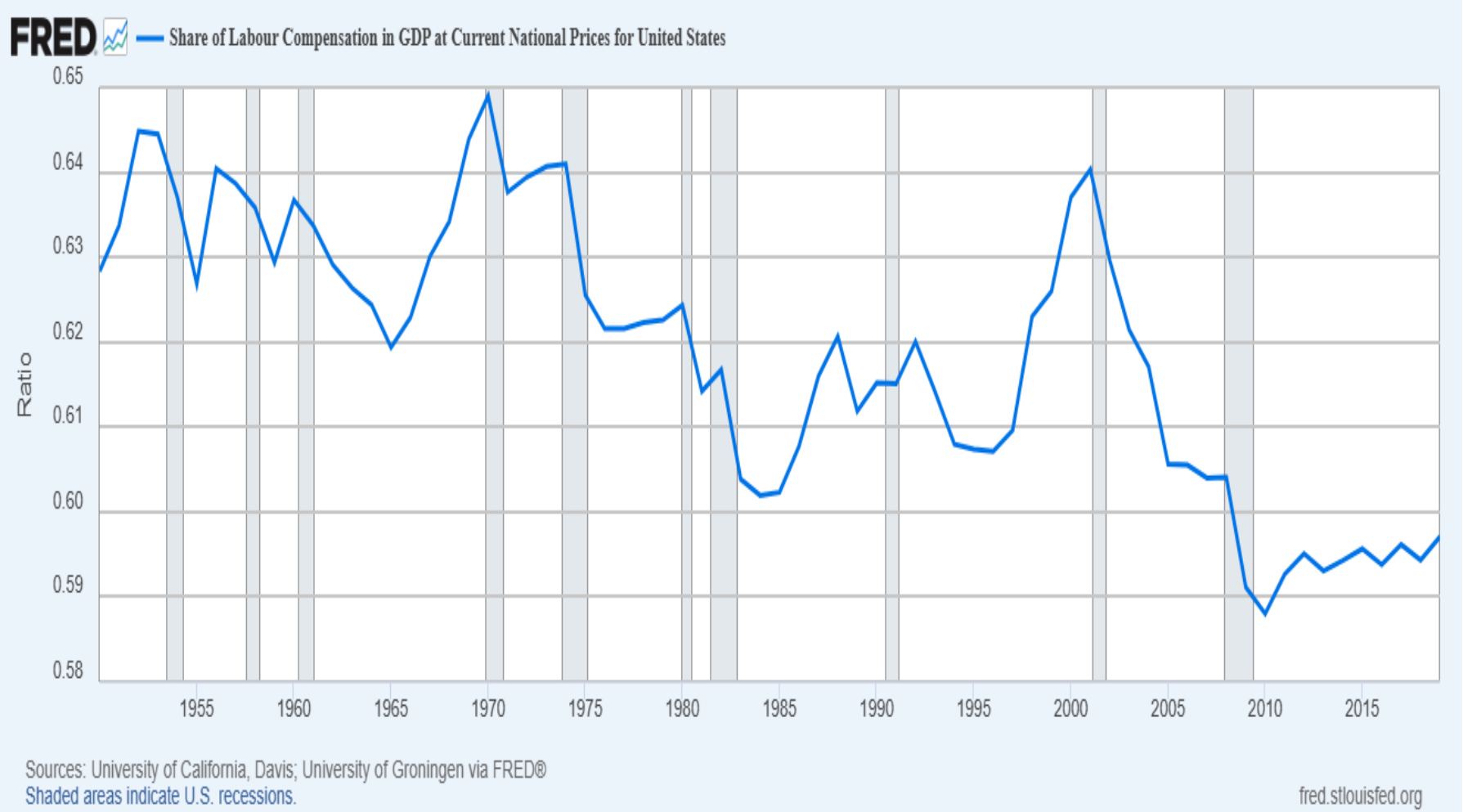

go back

# Capital to Output Ratio over Time



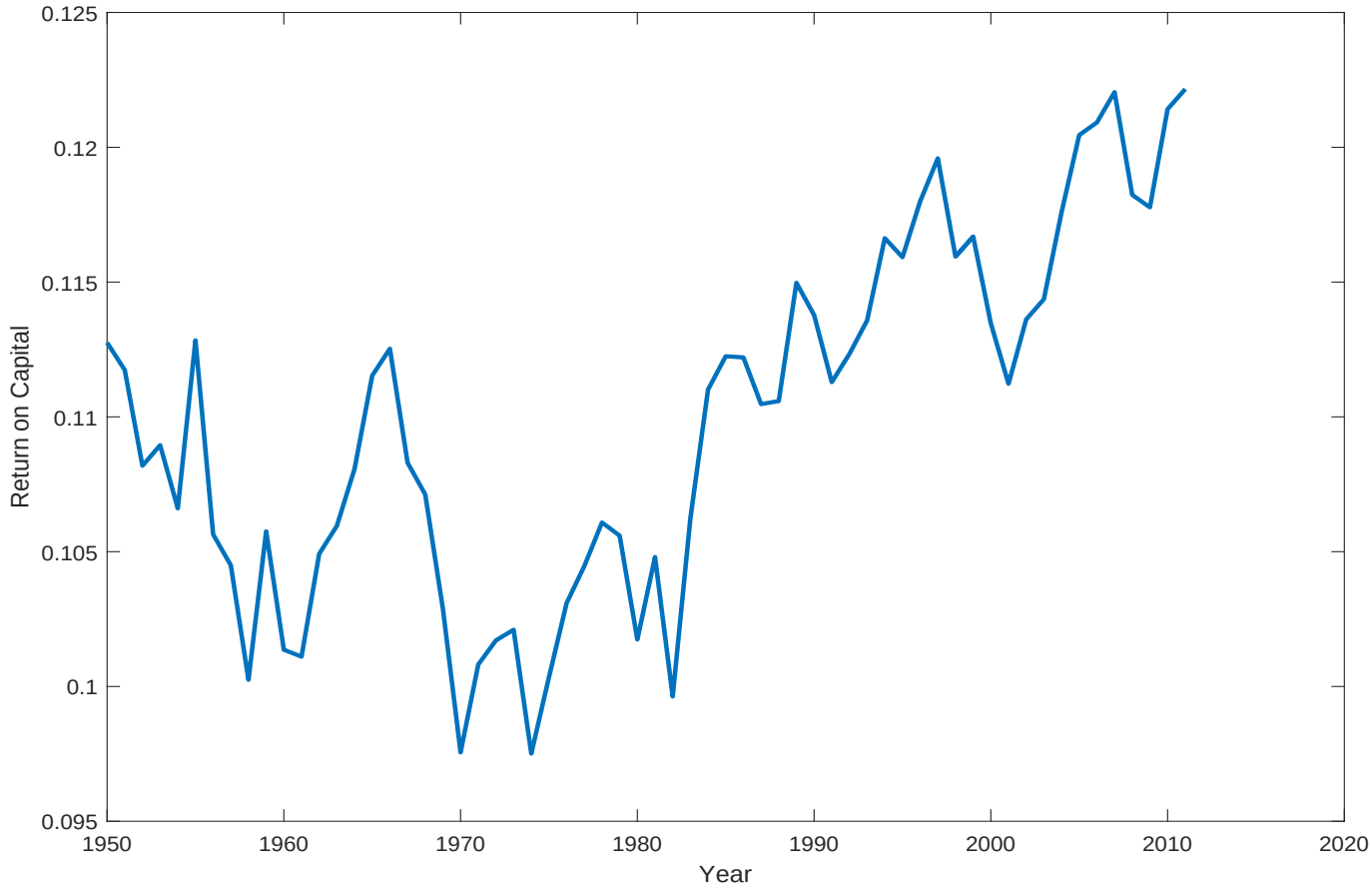

go back

# Labor Share over Time



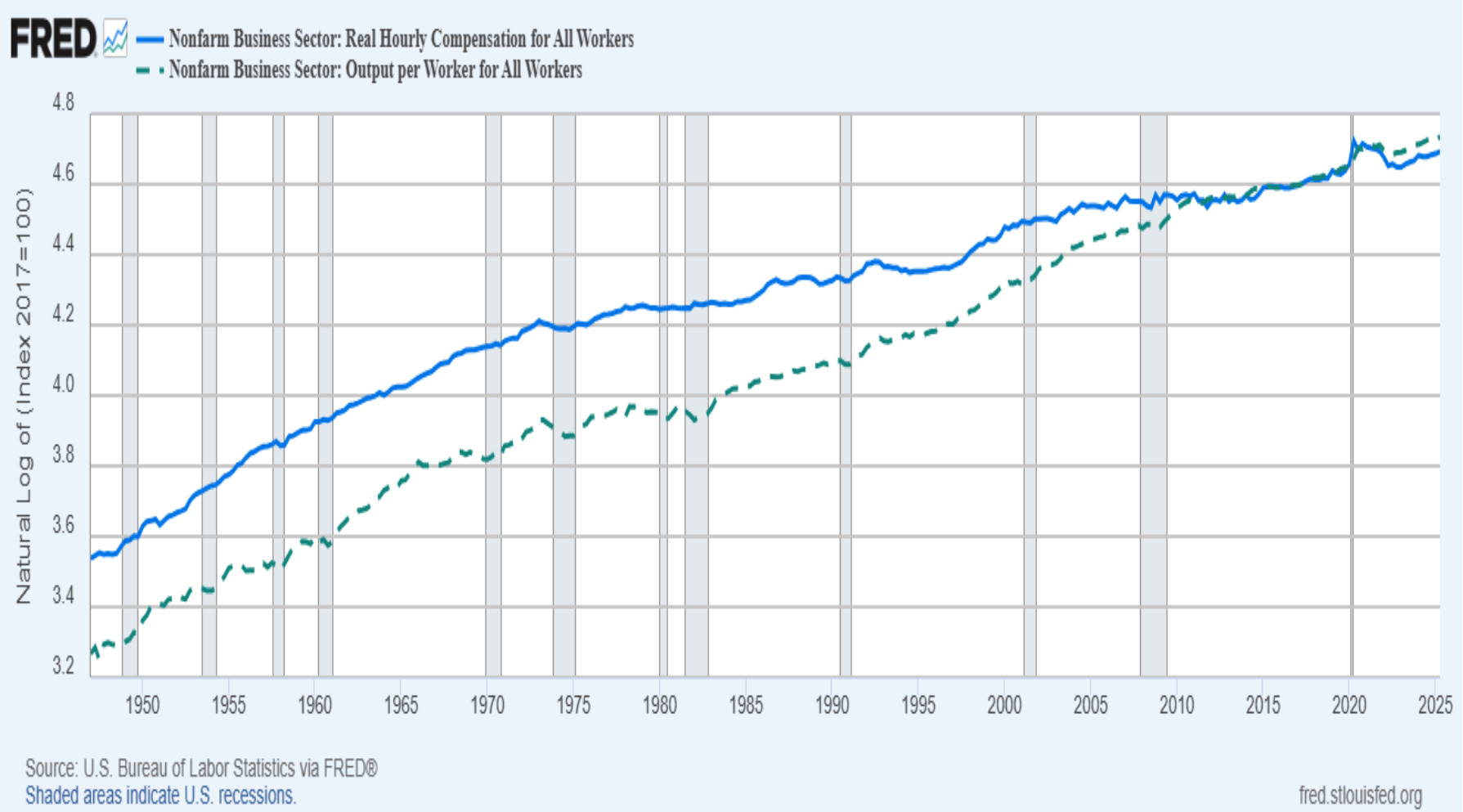

go back

# Return on Capital over Time




go back

# Real Wage over Time




go back

# Stylized Facts: Cross-Section

1. There are large differences in income per capita across countries ▸ table
2. There are some examples where poor countries catch up (growth miracles), otherwise where they do not (growth disasters) ▸ table
3. Human capital (e.g. education) strongly correlated with income per capita ▸ table

▸ jump ahead

# Income Differences

| | | GDP per Person |
|---|---|---|
| High income countries | | |
| | Canada | $35,180 |
| | Germany | $34,383 |
| | Japan | $30,232 |
| | Singapore | $59,149 |
| | United Kingdom | $32,116 |
| | United States | $42,426 |
| Middle income countries | | |
| | China | $8,640 |
| | Dominican Republic | $8,694 |
| | Mexico | $12,648 |
| | South Africa | $10,831 |
| | Thailand | $9,567 |
| | Uruguay | $13,388 |
| Low income countries | | |
| | Cambodia | $2,607 |
| | Chad | $2,350 |
| | India | $3,719 |
| | Kenya | $1,636 |
| | Mali | $1,157 |
| | Nepal | $1,281 |

go back

# Growth Miracles and Disasters

| | 1970 Income | 2011 Income | % increase (1970-2011) |
|---|---|---|---|
| | Growth Miracles | | |
| South Korea | $1918 | $27,870 | 1353 |
| Taiwan | $4,484 | $33,187 | 640 |
| China | $1,107 | $8,851 | 700 |
| Botswana | $721 | $14,787 | 1951 |
| | Growth Disasters | | |
| Madagascar | $1,321 | $937 | -29 |
| Niger | $1,304 | $651 | -50 |
| Burundi | $712 | $612 | -14 |
| Central African Republic | $1,148 | $762 | -34 |

go back

# Education and Income Per Capita


go back

# Solow Model

Solow model (Solow, 1953): used to study long-run growth and cross-country income differences

Does nice job with stylized facts

Main implication of model: productivity is key

- Productivity key to sustained growth (not factor accumulation)
- Productivity key to understanding cross-country income differences (not level of capital)

Model takes productivity to be exogenous. What is it? How to increase it?

# Model Basics

Time runs from $t$ (the present) onwards into infinite future

Representative household and representative firm

Everything real, one kind of good (fruit)

# Production Function

Production function:

$$Y_t = AF(K_t, N_t)$$

- $K_t$: capital. Must be itself produced, used to produce other stuff, does not get completely used up in production process
- $N_t$: labor
- $Y_t$: output
- $A$: productivity (exogenous); scales output given inputs

Think about output as units of fruit. Capital is stock of fruit trees. Labor is time spent picking from the trees

# Properties of Production Function

Both inputs necessary: $F(0, N_t) = F(K_t, 0) = 0$

Increasing in both inputs: $F_K(K_t, N_t) > 0$ and $F_N(K_t, N_t) > 0$

Concave in both inputs: $F_{KK}(K_t, N_t) < 0$ and $F_{NN}(K_t, N_t) < 0$

Constant returns to scale: $F(qK_t, qN_t) = qF(K_t, N_t)$

# Example Production Function: Cobb-Douglas

$$F(K_t, N_t) = K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

# Consumption and Investment

Fruit can either be eaten (consumption) or re-planted in the ground (investment)

Investment yields another tree (capital) with a one-period delay

A constant fraction of output is invested, $0 \leq s \leq 1$. "Saving rate" or "investment rate"

Means $1 - s$ of output is consumed

Resource constraint:

$$Y_t = C_t + I_t$$

# Labor & Capital

Abstract from endogenous labor supply – labor supplied inelastically

Current capital stock is exogenous – depends on past decisions

- We often refer to variables like this as state variables

Capital accumulation, $0 < \delta < 1$ depreciation rate:

$$K_{t+1} = I_t + (1 - \delta)K_t$$