Lecture 3: Capital Accumulation and the Steady State in the Solow Model

ECON 30020: Intermediate Macroeconomics

Prof. Eric Sims

University of Notre Dame

Spring 2026

Readings

GLS Ch. 5 (Solow Growth Model)

System of Equation

The basic Solow model can be reduced to a system of equations

These equations describe:

- Production and scarcity
- The accumulation process for capital
- Economic behavior

Equations of Model

$$Y_t = AF(K_t, N_t)$$

$$Y_t = C_t + I_t$$

$$K_{t+1} = I_t + (1 - \delta)K_t$$

$$I_t = sY_t$$

Four equations

Four endogenous variables (Y_t, C_t, I_t, K_{t+1})

Three exogenous variables (A, K_t, N_t). NOTE: K_t is predetermined within period, and is therefore exogenous

- Lack of subscript on A: if it changes (exogenously), that change is permanent (in expectation)

Central Equation

Equations can be combined into one:

$$K_{t+1} = sAF(K_t, N_t) + (1-\delta)K_t$$

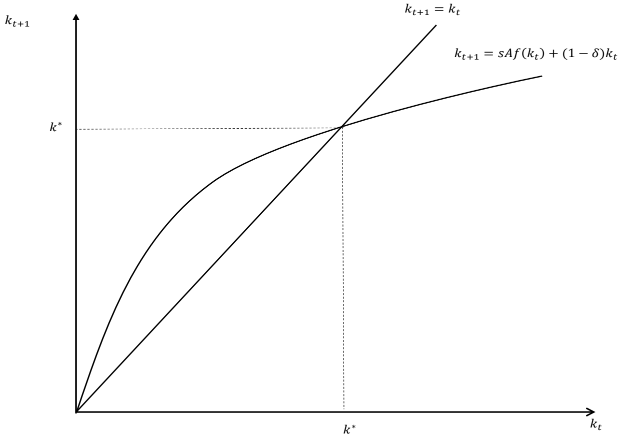
Define lowercase variables as "per worker." $k_t = \frac{K_t}{N_t}$. In per-worker terms:

$$k_{t+1} = sAf(k_t) + (1-\delta)k_t$$

One equation describing dynamics of k_t

Once you know dynamic path of capital, you can recover everything else

Plot of the Central Equation



The Steady State

The steady-state capital stock is the value of capital at which $k_{t+1} = k_t$. Label this k^*

Graphically, this is where the curve (the plot of k_{t+1} against k_t) crosses the 45-degree line (a plot of $k_{t+1} = k_t$)

Via assumptions of the production function along with auxiliary assumptions (the "Inada conditions"), there exists one non-zero steady-state capital stock

The steady state is "stable" in the sense that for any initial $k_t \neq 0$, the capital stock will converge to this point

"Once you get there, you sit there"

Since capital governs everything else, all other variables go to a steady state determined by k^*

Algebraic Example

Suppose $f(k_t) = k_t^\alpha$. Then:

$$
\begin{aligned}
k^* &= \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}} \\
y^* &= Ak^{*\alpha} \\
c^* &= (1-s)Ak^{*\alpha} \\
i^* &= sAk^{*\alpha}
\end{aligned}
$$

Factor Prices

Not central to understanding dynamics, but relevant for stylized facts

Suppose there exists one firm that hires labor and capital each period to maximize profit. Normalize price of output to 1

$$\max_{N_t, K_t} \quad AF(K_t, N_t) - w_t N_t - R_t K_t$$

w_t and R_t are (real) factor prices: wage and “rental rate.”

Factor Prices Equal Marginal Products

Factor prices equal marginal products:

$$w_t = AF_N(K_t, N_t)$$

$$R_t = AF_K(K_t, N_t)$$

With Cobb-Douglas production function:

$$w_t = (1-\alpha)AK_t^{\alpha}N_t^{-\alpha} = (1-\alpha)Ak_t^{\alpha}$$

$$R_t = \alpha AK_t^{\alpha-1}N_t^{1-\alpha} = \alpha Ak_t^{\alpha-1}$$

Dynamic Effects of Changes in Exogenous Variables

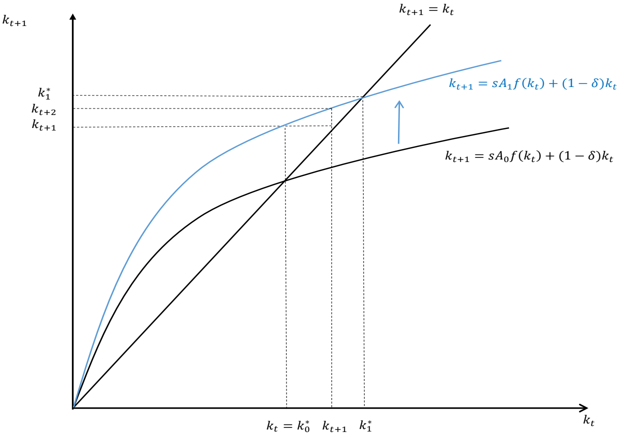
Want to consider the following exercises:

- What happens to endogenous variables in a dynamic sense after a permanent change in A^*
- What happens to endogenous variables in a dynamic sense after a permanent change in s (the saving rate)?

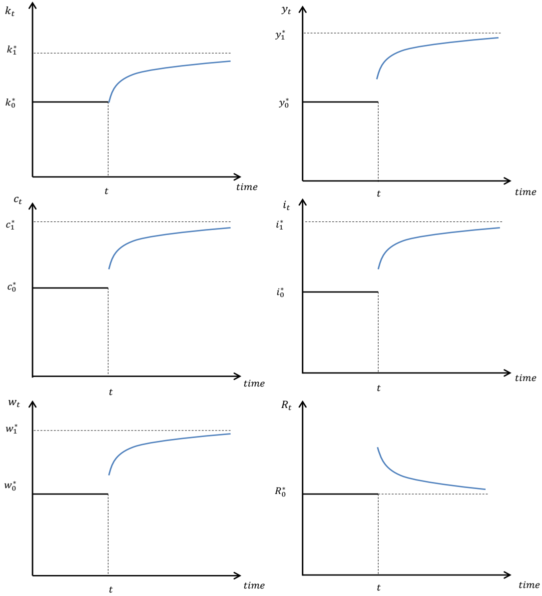
For these exercises:

1. Assume we start in a steady state
2. Graphically see how the steady state changes after the change in productivity or the saving rate
3. Current capital stock cannot change (it is predetermined/exogenous). But $k_t \neq k^*$. Use dynamic analysis of the graph to figure out how k_t reacts dynamically
4. Once you have that, you can figure out what everything else is doing

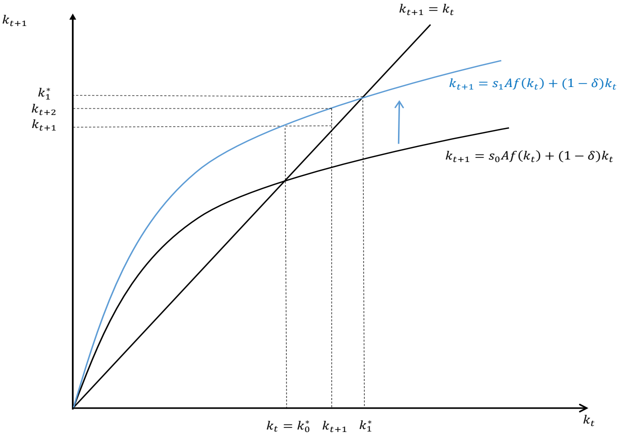
Permanent Increase in A



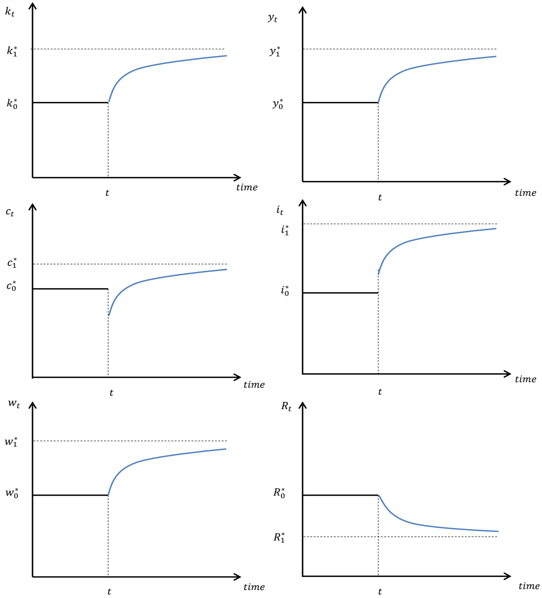
Impulse Response Functions: Permanent Increase in A



Permanent Increase in s



Impulse Response Functions: Permanent Increase in s



Quantitative Exercises

Assume Cobb-Douglas

Assign parameter and exogenous variable values:

$$A = 1,\ s = 0.2,\ \alpha = 1/3,\ \delta = 0.1$$

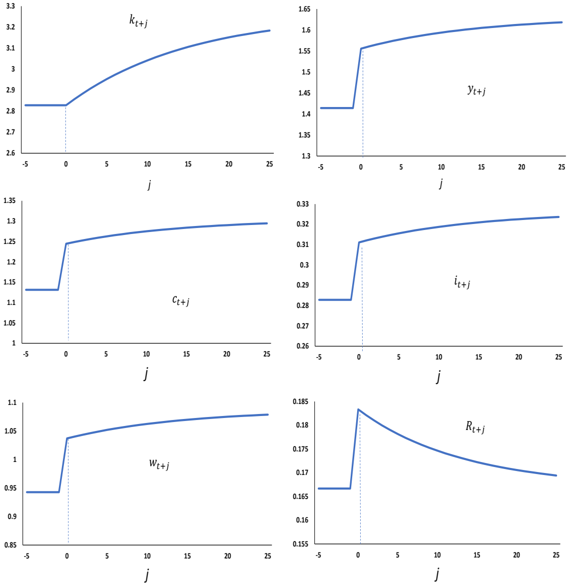
Solve for the numeric steady state

Assume economy sits in steady state from $t - N$ to $t - 1$, where $N = 5$.

In period t, assume A increase to 1.1.

In Excel, trace out paths of k_{t+j}, y_{t+j}, etc., for $j = 0, 1, ...$

Quantitative IRFs: Productivity Improvement



Discussion

Neither changes in A nor s trigger sustained increases in growth

Each triggers faster growth for a while while the economy accumulates more capital and transitions to a new steady state

In the long run, there is no growth in this model – it goes to a steady state!

We'll fix that. You can kind of see, however, that sustained growth must come from increases in productivity. Why?

- No limit on how high A can get – it can just keep increasing. Upper bound on s
- Repeated increases in s would trigger continual decline in R_t, inconsistent with stylized facts

The Bottom Line

Sustained growth must be due to productivity growth, not factor accumulation

You can't save your way to more growth

Key model assumption: diminishing returns to capital