Lecture 4: Golden Rule Saving and the Augmented Solow Model

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 5 (Solow Growth Model)

GLS Ch. 6 (Augmented Solow Model)

Optimal Saving Rate

What is the "optimal" saving rate, s?

Utility comes from consumption, not output

Higher s has two competing effects – the "size of the pie" and the "fraction of the pie":

- More capital $\rightarrow$ more output $\rightarrow$ more consumption (bigger size of the pie)
- Consume a smaller fraction of output $\rightarrow$ less consumption (eat a smaller fraction of the pie)

The Golden Rule

The Golden Rule saving rate: value of s that maximizes steady-state consumption, c^*

- $s = 0$: $c^* = 0$
- $s = 1$: $c^* = 0$

Implicitly characterized by $Af'(k^*) = \delta$. Graphical intuition.

Dynamic Inefficiency

Being "below" the Golden Rule does not necessarily mean that an economy is not saving enough

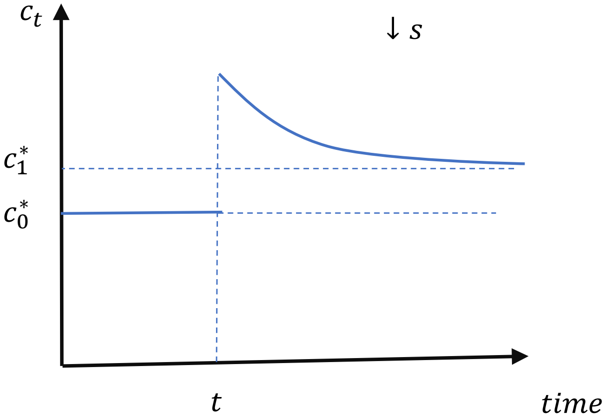
- There is a dynamic tradeoff: $\uparrow s$ today means less consumption today, but more in the future
- Whether that is good or not depends on how the future is valued relative to the present (i.e., discounting)

But being "above" the Golden Rule cannot be optimal. We say that it is dynamically inefficient

- By reducing the saving rate, could get more consumption both in present and in the future
- This wold be a Pareto improvement across all dates

Little or no evidence to suggest any modern economy is dynamically inefficient

Dynamic Inefficiency



Growth

Wrote down a model to study growth

But model converges to a steady state with no growth

Isn't that a silly model?

It turns out, no

Augmented Solow Model

Production function is:

$$Y_t = AF(K_t, Z_t N_t)$$

- Z_t: labor-augmenting productivity
- $Z_t N_t$: efficiency units of labor
- Assume Z_t and N_t both grow over time (initial values in period 0 normalized to 1):

$$Z_t = (1+z)^t$$
$$N_t = (1+n)^t$$

- $z = n = 0$: case we just did
- Z_t not fundamentally different from A_t. Convenient to use Z_t to control <u>growth</u> while A controls <u>level</u> of productivity

Per Efficiency Unit Variables

Define $\widehat{k}_t = \frac{K_t}{Z_t N_t}$ and similarly for other variables

- We do this because $\widehat{k}_t$ will be stationary (i.e., converge to a steady state)

Lower case variables: per-capita (or per-worker)

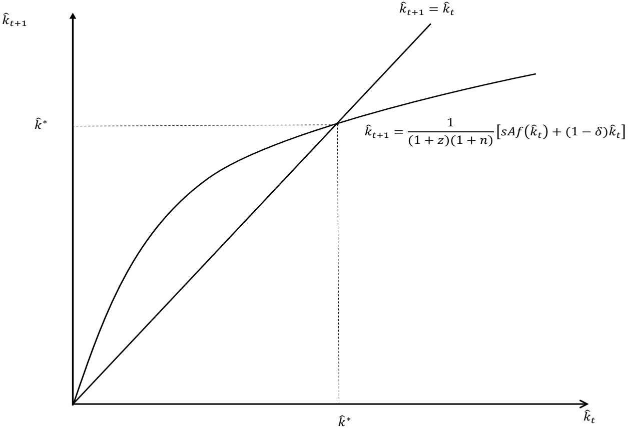
Lower case variables with "hats": per efficiency unit variables

Modified central equation of model is:

$$\widehat{k}_{t+1} = \frac{1}{(1+z)(1+n)} \left[sAf(\widehat{k}_t) + (1-\delta)\widehat{k}_t \right]$$

Practically the same as before, though in per efficiency units of capital

Plot of Modified Central Equation



Steady-State Growth I

Via similar arguments to earlier, there exists a steady state $\widehat{k}^*$ at which $\widehat{k}_{t+1} = \widehat{k}_t$. Economy converges to this point from any non-zero initial value of $\widehat{k}_t$

Economy converges to a steady state in which per efficiency unit variables do not grow. What about actual and per capita variables? If $\widehat{k}_{t+1} = \widehat{k}_t$, then:

$$
\begin{aligned}
\frac{K_{t+1}}{Z_{t+1}N_{t+1}} &= \frac{K_t}{Z_t N_t} \\
\frac{K_{t+1}}{K_t} &= \frac{Z_{t+1}N_{t+1}}{Z_t N_t} = (1+z)(1+n) \\
\frac{k_{t+1}}{k_t} &= \frac{Z_{t+1}}{Z_t} = 1+z
\end{aligned}
$$

Steady-State Growth II

Level of capital stock grows at approximately sum of growth rates of Z_t and N_t

Per-capita capital stock grows at rate of growth in Z_t

This growth is manifested in output and the real wage, but not the return on capital

Steady State Growth and Stylized Facts

Once in steady state, we have:

$$\frac{y_{t+1}}{y_t} = 1 + z$$

$$\frac{k_{t+1}}{k_t} = 1 + z$$

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{K_t}{Y_t}$$

$$\frac{w_{t+1} N_{t+1}}{Y_{t+1}} = \frac{w_t N_t}{Y_t}$$

$$R_{t+1} = R_t$$

$$\frac{w_{t+1}}{w_t} = 1 + z$$

These are the six time series stylized facts!