Lecture 7: A Two-Period Consumption-Saving Model

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 9

Microeconomics of Macro

We now move from the long run (decades and longer) to the medium run (several years) and short run (months up to several years)

In long run, we did not explicitly model most economic decision-making – just assumed rules (e.g., consume a constant fraction of income)

Building blocks of the remainder of the course are decision rules of optimizing agents and a concept of equilibrium

This is micro but in an aggregate context

Basic Framework

We will start by studying optimal decision rules

Then we work up to aggregation and equilibrium

Framework is dynamic but only two periods (t, the present, and $t+1$, the future)

Will also work with representative agents: one household and one firm

Unrealistic but useful abstraction and can be motivated in world with heterogeneity through insurance markets

Consumption

Consumption the largest expenditure category in GDP (60-70 percent)

Representative household receives exogenous amount of income in periods t and $t+1$

- Unless otherwise noted, assume no uncertainty, i.e., future endowment of income is known in period t

Household may consume or save/borrow – must decide how to divide its income in t between consumption and saving/borrowing

Everything real – think about one good as "fruit"

Basics

Income of Y_t and Y_{t+1}. Future income known with certainty

Consumes C_t and C_{t+1}

Begins life with no wealth, and can save $S_t = Y_t - C_t$ (can be negative, which is borrowing)

Earns/pays real interest rate, r_t, on saving/borrowing

Price-taker: takes r_t as given

Do not model a financial intermediary (e.g., a bank), but assume existence of option to borrow/save through this intermediary

Budget Constraints

Two flow budget constraints in each period:

$$C_t + S_t \leq Y_t$$
$$C_{t+1} + S_{t+1} - S_t \leq Y_{t+1} + r_t S_t$$

Saving vs. Savings: saving is a flow and savings is a stock. Saving is the change in the stock

As written, S_t and S_{t+1} are stocks

In period t, no distinction between stock and flow because no initial stock

$S_{t+1} - S_t$ is flow saving in period $t+1$; S_t is the stock of savings household takes from t to $t+1$, and S_{t+1} is the stock it takes from $t+1$ to $t+2$

$r_t S_t$: income earned on the stock of savings brought into $t+1$

Terminal Condition and the IBC

Household would not want $S_{t+1} > 0$. Why? There is no $t + 2$. Don't want to die with positive assets

Household would like $S_{t+1} < 0$ – die in debt. Lender would not allow that

Hence, $S_{t+1} = 0$ is a terminal condition

Assume budget constraints hold with equality, and eliminate S_t, leaving:

$$C_t + \frac{C_{t+1}}{1 + r_t} = Y_t + \frac{Y_{t+1}}{1 + r_t}$$

This is called the intertemporal budget constraint (IBC). Says that present discounted value of the stream of consumption equals the present discounted value of stream of income.

Preferences

Household gets utility from how much it consumes

Utility function: $u(C_t)$. "Maps" consumption into utils

Assume: $u'(C_t) > 0$ (positive marginal utility) and $u''(C_t) < 0$ (diminishing marginal utility)

"More is better, but at a decreasing rate"

Example

Example utility function:

$$u(C_t) = \ln C_t$$
$$u'(C_t) = \frac{1}{C_t} > 0$$
$$u''(C_t) = -C_t^{-2} < 0$$

Utility is completely <u>ordinal</u> – no meaning to magnitude of utility (it can be negative). Only useful to compare alternatives

Lifetime Utility

Lifetime utility is a weighted sum of utility from period t and $t+1$ consumption:

$$U = u(C_t) + \beta u(C_{t+1})$$

$0 < \beta < 1$ is the discount factor – it is a measure of how impatient the household is.

Household Problem

Technically, household chooses C_t and S_t in first period subject to the first-period flow constraint. This effectively determines C_{t+1} from the second-period flow constraint

Think instead about choosing C_t and C_{t+1} in period t subject to the IBC

$$\max_{C_t, C_{t+1}} \quad U = u(C_t) + \beta u(C_{t+1})$$

s.t.

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$$

Euler Equation

First order optimality condition is famous in economics – the "Euler equation" (pronounced "oiler")

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$

This is just an MRS = price ratio condition!

Necessary but not sufficient for optimality

Logic: at optimum, household is indifferent between consuming more today vs. saving and consumption principle plus interest tomorrow

Doesn't determine level of consumption. To do that need to combine with IBC

Indifference Curve

Think of C_t and C_{t+1} as different goods (different in time dimension)

Indifference curve: combinations of C_t and C_{t+1} yielding fixed overall level of lifetime utility

Different indifference curve for each different level of lifetime utility. Direction of increasing preference is "northeast"

Slope of indifference curve at a point is the negative ratio of marginal utilities (or marginal rate of substitution, MRS):

$$\text{slope} = -\frac{u'(C_t)}{\beta u'(C_{t+1})}$$

Given assumption of $u''(\cdot) < 0$, steep near origin and flat away from it

Budget Line

Graphical representation of IBC

Shows combinations of C_t and C_{t+1} consistent with IBC holding, given Y_t, Y_{t+1}, and r_t

- Points inside budget line: do not exhaust resources
- Points outside budget line: infeasible

By construction, must pass through point $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$ ("endowment point")

Slope of budget line is negative gross real interest rate:

$$\text{slope} = -(1 + r_t)$$

Optimality

Objective is to choose a consumption bundle on highest possible indifference curve that does not violate IBC

At this point, indifference curve and budget line are tangent (which is same condition as Euler equation)

Optimality: Graphically