Problem Set 3

ECON 30020, Intermediate Macroeconomics, Spring 2026
The University of Notre Dame
Professor Sims

Instructions: Please prepare legible, complete solutions to the following problems. You may work with others, and consult whatever resources you wish, but you are responsible for your own work and must turn in your own assignment. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers and/or figures from Excel – you need not turn in Excel code. An image/scan of the problem set should be uploaded onto the Canvas site via the "assignments" tab no later than 5:00 pm on the due date of March 3.

1. **Consumption Under Uncertainty**: Suppose that a household lives for two periods. It receives a flow of income of $Y_t = 2$ in the first period; this is deterministic. In the second period, it receives a flow of income of $Y_{t+1}(1) = 3$ with probability 1/2 (call this state 1) and a flow of income of $Y_{t+1}(2) = 1$ with probability 1/2 (call this state 2). It can save or borrow in the first period at interest rate r_t. The period t budget constraint is:

$$C_t + S_t = Y_t$$

The second-period budget constraint must hold in both states of the world, so we have:

$$C_{t+1}(1) = Y_{t+1}(1) + (1 + r_t)S_t$$
$$C_{t+1}(2) = Y_{t+1}(2) + (1 + r_t)S_t$$

The household wants to maximize the expected value of lifetime utility:

$$U = u(C_t) + \beta \mathbb{E}u(C_{t+1}) = u(C_t) + \frac{1}{2}\beta u(C_{t+1}(1)) + \frac{1}{2}\beta u(C_{t+1}(2))$$

You may assume that $\beta(1 + r_t) = 1$.

   (a) Derive the consumption Euler equation.

   (b) Explain, in words, why even though $\beta(1 + r_t) = 1$, consumption may not be constant in expectation across periods.

   (c) Suppose that the flow utility function is the natural log, i.e., $u(C_t) = \ln C_t$. Assume that $\beta = 0.95$ (and, therefore, $1 + r_t = 0.95^{-1}$). Solve for the optimal value of saving, S_t, as well as consumption in the first period, expected consumption in the second period, and consumption in both states of the world in the second period.

   (d) Re-do this analysis, but suppose that $Y_{t+1}(1) = 4$ and $Y_{t+1}(2) = 0$. What happens to saving? Explain briefly.

2. **Taxes, Transfers, and Efficiency:** Suppose an economy is populated by two types of agents: type 1 agents are "poor" and type 2 agents are "rich." There are $N_1 = N_2$ types of each agent. Each type of agent behaves as a price taker. They have identical preferences with the same discount factor, β. Time lasts for two periods. Type 1 agents receive an endowment pattern of $Y_t(1) = Y_{t+1}(1) = 1/2$. Type 2 agents receive an endowment pattern of $Y_t(2) = Y_{t+1}(2) = 3/2$. Each type of agent pays a tax (or receives a transfer) from the government in period t, $T_t(j)$. There are no taxes or transfers in period $t+1$. Each type of agent ($j = 1$ or $j = 2$) solve the following problem:

$$\max_{C_t(j), S_t(j)} \quad U(j) = \ln C_t(j) + \beta \ln C_{t+1}(j)$$

s.t.

$$C_t(j) + S_t(j) = Y_t(j) - T_t(j)$$
$$C_{t+1}(j) = Y_{t+1}(j) + (1 + r_t)S_t(j)$$

The government runs a balanced budget. That is, if it taxes one type of agent, it must transfer those resources to the other agent. The government budget constraint in period t is therefore:

$$N_1 T_t(1) + N_2 T_t(2) = 0$$

(a) Solve for the Euler equation for agents of type $j = 1$ and $j = 2$.

(b) Solve for the consumption function of agents of type $j = 1$ and $j = 2$ for arbitrary values of taxes and transfers.

(c) What is the aggregate market-clearing condition for this economy? Use this to solve for the competitive equilibrium real interest rate. Argue that the equilibrium interest rate and aggregate consumption are independent of the values of taxes and transfers.

(d) Suppose that we initially have $T_t(1) = T_t(2) = 0$ (i.e., the government does no redistribution). Assume $\beta = 0.95$. Given your answer above, solve for the individual values of consumption for both types of agents in both periods. Calculate $U(1)$ and $U(2)$.

(e) Suppose that the government wants to pick $T_t(1)$ and $T_t(2)$ to maximize the following social welfare function:

$$\mathbb{W} = N_1 U_1 + N_2 U_2$$

The optimization is subject to the equilibrium conditions derived above (i.e., the consumption function) and the balanced budget constraint. Solve for the optimal $T_t(1)$ and $T_t(2)$, and calculate the lifetime utilities of the two types of agents under this tax and transfer scheme.

(f) Based on your answers to this question, comment on the following statement: "Under certain conditions, a competitive equilibrium allocation is efficient, but it may not be desirable."

3. **Debt vs. Equity Financing of Firm Capital:** In class, we assumed that a representative firm had to borrow to finance its expenditure on new capital. Assume a Cobb-Douglas production function. With debt finance, the firm's problem is:

$$\max_{N_t, I_t} \quad V_t = D_t + \frac{1}{1+r_t} D_{t+1}$$

s.t.

$$K_{t+1} = I_t + (1-\delta)K_t$$
$$D_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t$$
$$D_{t+1} = A_{t+1} K_{t+1}^{\alpha} N_{t+1}^{1-\alpha} + (1-\delta)K_{t+1} - w_{t+1} N_{t+1} - (1+r_t^I) I_t$$

(a) Solve for the optimal first-order conditions.

(b) Now, instead suppose that the firm uses equity instead of debt. In particular, it buys the capital directly in period t, which comes out of the period-t dividend. And it does not have to pay back any interest nor principal on a loan. The firm's problem in this case is:

$$\max_{N_t, I_t} \quad V_t = D_t + \frac{1}{1+r_t} D_{t+1}$$

s.t.

$$K_{t+1} = I_t + (1-\delta)K_t$$
$$D_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - I_t$$
$$D_{t+1} = A_{t+1} K_{t+1}^{\alpha} N_{t+1}^{1-\alpha} + (1-\delta)K_{t+1} - w_{t+1} N_{t+1}$$

Note: we are implicitly assuming a one-to-one transformation between output, investment, and capital: one unit of unconsumed output turns into one unit of future capital. This means that the price of capital (relative to output) is one. Solve for the first-order conditions for the firm.

(c) Under what condition are the first-order conditions from the two versions of the problem (i.e., equity vs. debt) the same? Explain briefly.

Now, let's instead suppose that the household owns the capital stock, makes investment decisions, and leases capital to the representative firm on a period-by-period basis. Each period, the firm just hires labor at w_t and rents capital at R_t. Its problem is:

$$\max_{N_t, K_t} \quad V_t = D_t + \frac{D_{t+1}}{1+r_t}$$

s.t.

$$D_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t K_t$$
$$D_{t+1} = A_{t+1} K_{t+1}^{\alpha} N_{t+1}^{1-\alpha} - w_{t+1} N_{t+1} - R_{t+1} K_{t+1}$$

(d) Derive the optimality conditions for the firm.

The household makes the capital accumulation decision. It faces two flow budget constraints:

$$C_t + S_t - S_{t-1} + I_t \leq w_t N_t + R_t K_t + D_t + r_{t-1} S_{t-1}$$
$$C_{t+1} + S_{t+1} - S_t + I_{t+1} \leq w_{t+1} N_{t+1} + R_{t+1} K_{t+1} + D_{t+1} + r_t S_t$$

Where:

$$K_{t+1} = I_t + (1-\delta) K_t$$
$$K_{t+2} = I_{t+1} + (1-\delta) K_t$$

Assume that the household begins with an exogenously given, non-zero amount of physical capital, $K_t > 0$. Assume that the household begins with no financial assets, $S_{t-1} = 0$.

(e) What are the terminal conditions for the two assets the household can hold? Explain briefly? Imposing those terminal conditions, derive the intertemporal budget constraint.

Assume the household supplies a fixed amount of labor each period, $N_t = N_{t+1} = 1$. Its lifetime utility function is:

$$U = \ln C_t + \beta \ln C_{t+1}$$

The household can choose C_t, C_{t+1}, S_{t+1}, I_t, and I_{t+1}. It takes N_t and N_{t+1} as given.

(f) Find the first-order conditions for the household problem.

(g) What is the relationship between the first-order condition for the capital stock in the household's problem here compared to the setup where the firm owns capital and finances it via equity? What about with the setup where the firm finances capital with debt?

4. **Labor and Consumption Taxes:** Suppose there is a representative household that lives for two periods. The household potentially pays a consumption tax and a labor income tax. The two flow budget constraints facing the household are:

$$(1+\tau_t^C) C_t + S_t \leq (1-\tau_t^w) w_t N_t + D_t - T_t$$
$$(1+\tau_{t+1}^C) C_{t+1} + S_{t+1} - S_t \leq (1-\tau_{t+1}^w) w_{t+1} N_{t+1} + D_{t+1} - T_{t+1} + r_t S_t$$

τ^C and τ^w are proportional taxes on consumption expenditure and labor income, respectively. D_t and D_{t+1} are dividend flows from ownership in a representative firm. T_t and T_{t+1} are lump-sum taxes. The household takes dividends, lump-sum taxes, and tax rates as given. I am assuming that the household begins with no assets

The household's objective function is:

$$U = u(C_t, 1 - N_t) + \beta u(C_{t+1}, 1 - N_{t+1})$$

The utility function has the usual properties.

(a) Imposing the terminal condition on end-of-life household assets, derive the first-order optimal conditions for the household problem.

(b) Suppose that the consumption tax is constant, $\tau_t^C = \tau_{t+1}^C = \tau^C$. In this case, argue that the consumption tax does not directly affect the Euler equation for optimal consumption-saving. Argue that, in this case, the consumption tax is isomorphic to a labor income tax. Can you provide any intuition for that answer?