Lecture 10: A First Peak at Equilibrium – Endowment Economy

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 11

Motivation

We were just studying an optimal decision problem for a household

The outcome of which was an optimal decision rule (the consumption function)

The decision rule takes prices as given

In the two-period consumption-saving model, the only price is r_t (the real interest rate)

Modes of Economic Analysis

1. Decision Theory: derivation of optimal decision rules taking prices as given
   - e.g., consumption function, labor demand function, etc.
2. Partial Equilibrium: determine price that clears one market, taking prices in all other markets as given
   - e.g., the wage and the labor market, the interest rate and loanable funds market
3. General Equilibrium: simultaneously determine all prices to simultaneously clear all markets
   - e.g., simultaneously determine wage and interest rate to clear the labor and loanable funds markets

Macro is really about general equilibrium

Competitive Equilibrium

Webster's dictionary defines the word equilibrium to be "a state in which opposing forces or actions are balanced so that one is not stronger or greater than the other."

In economics, no agent has an incentive to change his or her behavior given the behavior of other agents and the constraints all agents face

Concretely: prices adjust to clear markets given optimizing behavior by agents (i.e., demand = suply)

Competitive Equilibrium: prices and allocations such that

(i) all agents are behaving according to optimal decision rules, **taking all other prices and behaviors as given**

(ii) all markets simultaneously clear

Endowment Economy

Endowment economy: no endogenous production. Income is just an exogenous endowment

Essentially, this fixes supply and it becomes particularly clear how prices adjust to clear markets

Two-period consumption-saving model:

- Optimal decision rule: consumption function
- Market: market for saving
- Price: r_t
- Market-clearing: in aggregate, saving is zero (equivalently, $Y_t = C_t$)
- Allocations: C_t, S_t, and C_{t+1} (i.e., optimal decision rules using the market-clearing r_t)

Environment

There are L total agents (L is "large") with identical preferences, but potentially different incomes. Index by $j = 1, \ldots, L$

Each can borrow/save at r_t and solves problem:

$$\max_{C_t(j), C_{t+1}(j)} \quad U(j) = u(C_t(j)) + \beta u(C_{t+1}(j))$$

s.t.

$$C_t(j) + \frac{C_{t+1}(j)}{1+r_t} = Y_t(j) + \frac{Y_{t+1}(j)}{1+r_t}$$

Optimal decision rule:

$$C_t(j) = C^d\left(Y_t(j), Y_{t+1}(j), r_t\right)$$

Or:

$$S_t(j) = Y_t(j) - C_t(j) = S^d\left(Y_t(j), Y_{t+1}(j), r_t\right)$$

Market-Clearing

What does it mean for markets to clear?

Aggregate saving must be equal to zero:

$$S_t = \sum_{j=1}^{L} S_t(j) = 0$$

Why? One agent's saving must be another's borrowing and vice-versa:

$$\sum_{j=1}^{L} (Y_t(j) - C_t(j)) = 0 \Rightarrow \sum_{j=1}^{L} Y_t(j) = \sum_{j=1}^{L} C_t(j)$$

In other words, aggregate income must equal aggregate consumption:

$$Y_t = C_t$$

Simple Case: Everyone Identical

Suppose agents all have identical endowment streams

Convenient to normalize $L = 1$ ("representative agent")

- Means average = aggregate
- Can drop j reference

Market-clearing is then:

$$Y_t = C^d\left(Y_t, Y_{t+1}, r_t\right)$$

Basic idea, find the r_t (endogenous) where this equation holds, taking endowments (Y_t and Y_{t+1}, exogenous) as given

Graphical Analysis

Can easily solve the above algebraically

Helpful to do so graphically

Complication: Y_t is on both the left- and right-hand sides

Solution:

- Define Y_t^d as desired expenditure
- Plot Y_t^d as a function of Y_t, taking all else as given
- Plot a 45-degree line (i.e., $Y_t^d = Y_t$), see if and where they intersect

Keynesian Cross and the IS Curve

Keynesian Cross: plot of <u>desired expenditure</u> (Y_t^d) against income (Y_t)

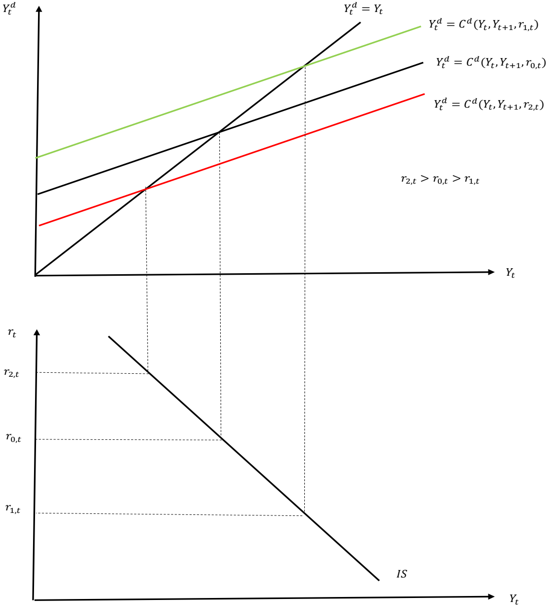
Autonomous expenditure: desired expenditure if zero income (i.e., $C^d(0, Y_{t+1}, r_t)$. This is the vertical intercept, assume greater than zero

Plot will be upward-sloping with slope <u>less than one</u> (i.e., $0 < MPC < 1$)

Will cross 45-degree line (slope of 1, points where $Y_t^d = Y_t$) exactly once

<u>IS curve</u> (investment = saving): set of (r_t, Y_t) pairs where everyone behaves optimally and income equals expenditure (equivalently, saving equals investment, or in an endowment economy, saving equals zero)

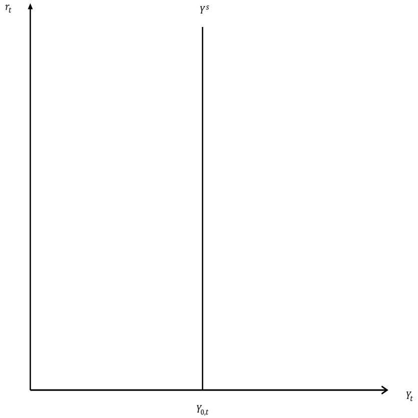
Derivation of the IS Curve



The Y^s Curve

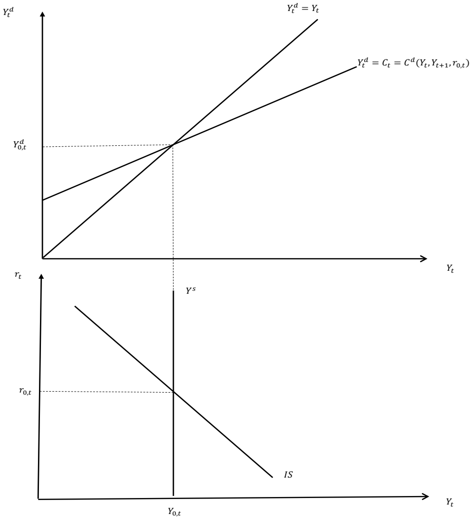
The Y^s curve summarizes the production side of the economy

In an endowment economy, there is no production! So the Y^s curve is just a vertical line at the exogenously given level of Y_t

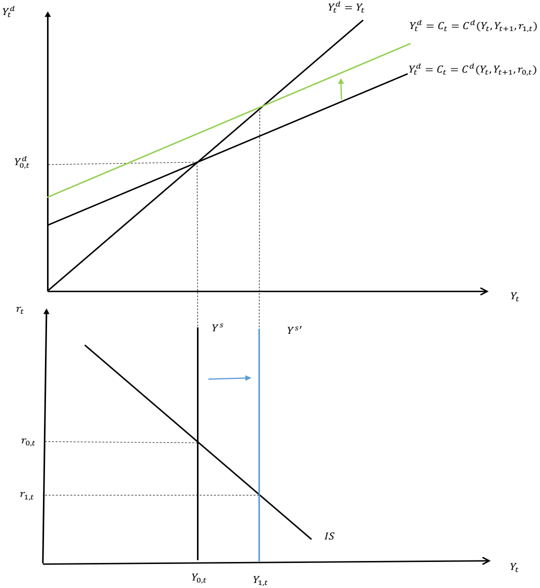


Equilibrium

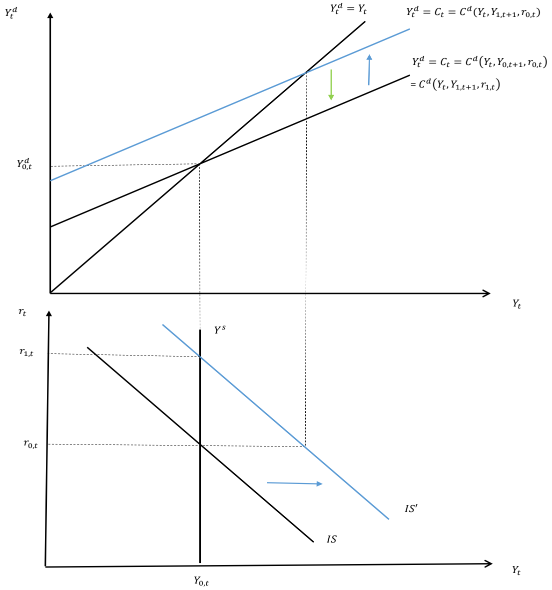
Must have income = expenditure (demand side) = production (supply-side). Find the r_t where IS and Y^s cross



Supply Shock: $\uparrow Y_t$



Demand Shock: $\uparrow Y_{t+1}$



Discussion

Market-clearing requires $C_t = Y_t$

For a given r_t, household does not want $C_t = Y_t$. Wants to smooth consumption relative to income

But in equilibrium cannot smooth without aggregative saving

r_t adjusts so that household is content to have $C_t = Y_t$

r_t ends up being a measure of how plentiful the future is expected to be relative to the present

Algebraic Example with Log Utility

With log utility, equilibrium real interest rate comes out to be (just take Euler equation and set $C_t = Y_t$ and $C_{t+1} = Y_{t+1}$)

$$1 + r_t = \frac{1}{\beta} \frac{Y_{t+1}}{Y_t}$$

r_t proportional to expected income growth

Potentially useful for thinking of "problem" of low real rates in last decades

Agents with Different Endowments

Suppose there are two types of agents, 1 and 2. L_1 and L_2 of each type

Identical preferences

Type 1 agents receive $Y_t(1) = 1$ and $Y_{t+1}(1) = 0$, whereas type 2 agents receive $Y_t(2) = 0$ and $Y_{t+1}(2) = 1$

Assume log utility, so consumption functions for each type are:

$$C_t(1) = \frac{1}{1+\beta}$$
$$C_t(2) = \frac{1}{1+\beta}\frac{1}{1+r_t}$$

Aggregate income in each period is $Y_t = L_1$ and $Y_{t+1} = L_2$

Equilibrium

With this setup, the equilibrium real interest rate is:

$$1 + r_t = \frac{1}{\beta} \frac{L_2}{L_1}$$

Noting that $L_2 = Y_{t+1}$ and $L_1 = Y_t$, this is the same as in the case where everyone is the same!

In particular, given aggregate endowments, equilibrium r_t does not depend on distribution across agents, only depends on aggregate endowment

Amount of income heterogeneity at micro level doesn't matter for macro outcomes. Example of "market completeness" and motivates studying representative agent problems more generally

- This would not hold if there were impediments to agents borrowing/saving