Lecture 12: Production, Investment, and Labor Demand

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 12

Production and Variable Labor

Two-period, optimizing, equilibrium model of the economy

- We model production, investment, and variable labor input
- Will focus on labor demand now; will return to labor supply soon

The production side is similar to the Solow model

Firm

There exists a representative firm. The firm produces output using capital, K_t, and labor, N_t, according to the following production function:

$$Y_t = A_t F(K_t, N_t)$$

A_t is exogenous productivity variable. Abstract from trend growth. Keep time subscript: want to distinguish between A_t and A_{t+1}

$F(\cdot)$ has the same properties as assumed in the Solow model – increasing in both arguments, concave in both arguments, both inputs necessary. For example:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

Capital Accumulation

Slightly differently than the Solow model, we assume that the firm makes the capital accumulation decision

Firm must borrow from a financial intermediary in order to finance its investment

- "Equity" versus "debt" finance would be equivalent absent financial frictions, which we will model
- Ownership of capital wouldn't make a difference absent financial frictions either (i.e., firm makes capital accumulation decision vs. household owning capital and leasing it to firms)

Current capital, K_t, is predetermined and hence exogenous. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

Prices Relevant for the Firm

Firm hires labor in a competitive market at (real) wage w_t (and w_{t+1} in the future)

Firm borrows to finance investment at

$$r_t^I = r_t + f_t$$

r_t^I is the interest rate relevant for the firm, while r_t is the interest rate relevant for the household

f_t is (an exogenous) variable representing a financial friction. We will refer to this as a credit spread

During financial crises observed credit spreads rise significantly

Dividend

The representative household owns the firm. The firm returns any difference between revenue and cost to the household each period in the form of a dividend

The household takes dividends as given (i.e., management and ownerships are distinct)

In period t, dividend is simply output less cost of labor (since borrowing cost of investment is borne in future)

$$D_t = Y_t - w_t N_t$$

Future Dividend and Terminal Condition

Terminal condition for the firm: firm wants $K_{t+2} = 0$ (die with no capital). This implies $I_{t+1} = -(1-\delta)K_{t+1}$, which we can think of as the firm "liquidating" its remaining capital after production in $t+1$

Who buys the liquidated capital? The household, who consumes ("eats") it

Liquidation is an additional source of revenue for the firm in $t+1$. Firm must also pay interest plus principal on its borrowing for investment in t:

$$D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1} - (1+r_t^I)I_t$$

Firm Valuation

Value of the firm: PDV of flow of dividends:

$$V_t = D_t + \frac{1}{1 + r_t} D_{t+1}$$

The relevant interest rate for discounting future profit is r_t, not r_t^l

This is because household owns the firm and discounts future dividend according to interest rate relevant to it

Firm Problem

Firm problem is to pick N_t and I_t to maximize V_t subject to accumulation equation:

$$\max_{N_t, I_t} \; V_t = D_t + \frac{1}{1 + r_t} D_{t+1}$$

s.t.

$$K_{t+1} = I_t + (1 - \delta) K_t$$

$$D_t = A_t F(K_t, N_t) - w_t N_t$$

$$D_{t+1} = A_{t+1} F(K_{t+1}, N_{t+1}) + (1 - \delta) K_{t+1} - w_{t+1} N_{t+1} - (1 + r_t^I) I_t$$

Firm can also pick N_{t+1}, but this is a static choice and doesn't depend on whether we think about the firm doing this in period t or $t + 1$

First-Order Conditions

$$w_t = A_t F_N(K_t, N_t)$$

$$1 + r_t^l = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1 - \delta)$$

Intuition: MB = MC

Wage condition exactly same as Solow model expression for wage

Investment condition can be re-written in terms of earlier notation by defining $R_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$ and:

$$R_{t+1} = r_t^l + \delta = r_t + f_t + \delta$$

Return on capital, R_{t+1}, closely related to real interest rate, r_t

Diversion: Debt vs. Equity Finance

We are assuming firm finances investment via debt. Equity finance:

$$D_t = Y_t - w_t N_t - I_t$$

$$D_{t+1} = Y_{t+1} + (1-\delta)K_{t+1} - w_{t+1}N_{t+1}$$

Debt: lower dividend in future. Equity: lower dividend in present

FOC w/ equity:

$$1 + r_t = A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta)$$

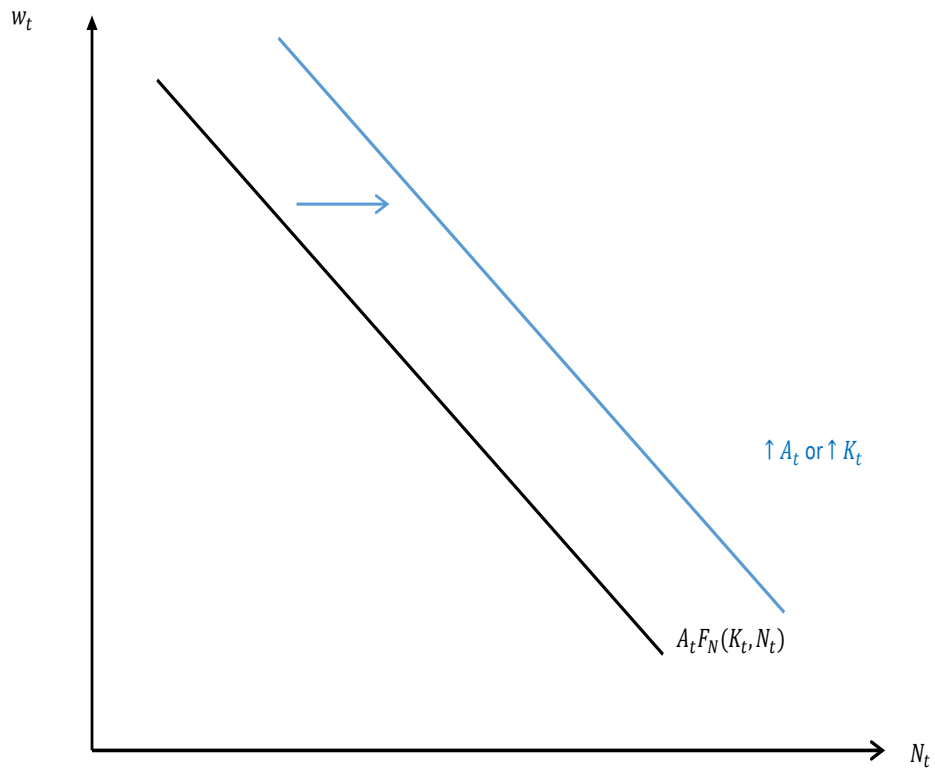
Firm would prefer equity if $f_t > 0$; otherwise, firm is indifferent (Modigliani-Miller 1958)

We assume some underlying friction prevents firm's ability to issue debt

Labor Demand

Labor FOC implicitly characterizes a downward-sloping labor demand curve:

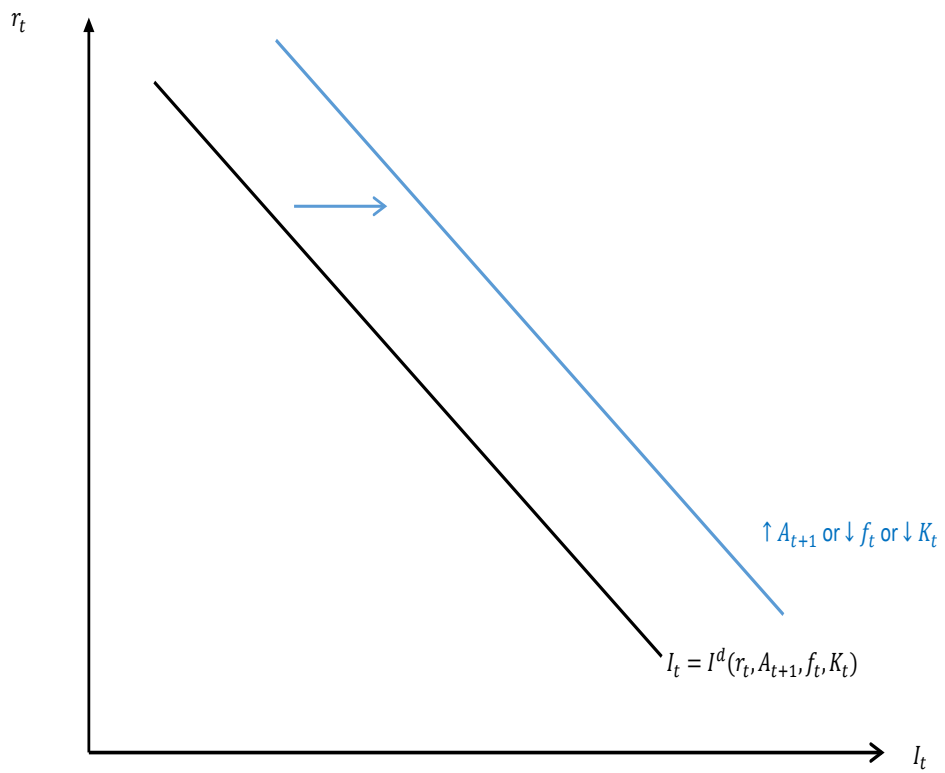
$$N_t = N^d(\underset{-}{w_t}, \underset{+}{A_t}, \underset{+}{K_t})$$



Investment Demand

Second first-order condition implicitly defines a demand for K_{t+1}, which can be used in conjunction with the accumulation equation to get an investment demand curve:

$$I_t = I^d(\underset{-}{r_t}, \underset{+}{A_{t+1}}, \underset{-}{f_t}, \underset{-}{K_t})$$



Financial Intermediary

Will not go into great detail

In period t, takes in deposits, S_t, from household; issues loans in amount I_t to firm

Pays r_t for deposits, and earns $r_t^I = r_t + f_t$ on loans

f_t is exogenous, and $f_t > 0$ means intermediary earns profit in $t+1$, which is returned to household as dividend:

$$D_{t+1}^I = (r_t + f_t) I_t - r_t S_t$$

Household takes <u>all</u> dividend payments (D_t, D_{t+1}, and D_{t+1}^I as given)