Lecture 9: Multiple Periods, Permanent Income Hypothesis, Uncertainty

ECON 30020: Intermediate Macroeconomics

Prof. Eric Sims

University of Notre Dame

Spring 2026

Readings

GLS Ch. 9-10

Multi-Period Extension: Utility

Suppose t is the present and time runs forward to $t + T$, where $T \geq 1$

Lifetime utility:

$$U = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots + \beta^T u(C_{t+T})$$

This is a form of geometric discounting: discounting between periods is always constant $(0 < \beta < 1)$

Multi-Period Extension: Budget Constraints

Flow budget constraint must hold each period:

$$C_t + S_t \leq Y_t$$

$$C_{t+1} + S_{t+1} - S_t \leq Y_{t+1} + r_t S_t$$

$$C_{t+2} + S_{t+2} - S_{t+1} \leq Y_{t+2} + r_{t+1} S_{t+1}$$

$$\vdots$$

$$C_{t+T} + S_{t+T} - S_{t+T-1} \leq Y_{t+T} + r_{t+T-1} S_{t+T-1}$$

$S_{t+j} - S_{t+j-1}$ for $j > 0$ is flow saving, whereas S_{t+j} is stock savings

Terminal condition: $S_{t+T} = 0$

Intertemporal Budget Constraint

Simplifying assumption: $r_{t+j} = r$ for any j

Present discounted value of stream of consumption must equal present discounted value of stream of income:

$$C_t + \frac{C_{t+1}}{1+r} + \frac{C_{t+2}}{(1+r)^2} + \cdots + \frac{C_{t+T}}{(1+r)^T} = Y_t + \frac{Y_{t+1}}{1+r} + \frac{Y_{t+2}}{(1+r)^2} + \cdots + \frac{Y_{t+T}}{(1+r)^T}$$

Euler Equation(s)

Will have T Euler equations for all adjacent periods of time:

$$\begin{aligned} u'(C_t) &= \beta(1+r)u'(C_{t+1}) \\ u'(C_{t+1}) &= \beta(1+r)u'(C_{t+2}) \\ &\vdots \\ u'(C_{t+T-1}) &= \beta(1+r)u'(C_{t+T}) \end{aligned}$$

- $\beta(1+r) > 1$: consumption increases
- $\beta(1+r) < 1$: consumption decreases
- $\beta(1+r) = 1$: consumption constant

Consumption Function

Assume $\beta(1+r) = 1$. Consumption function is:

$$C = \underbrace{\frac{r}{1 + r - \left(\frac{1}{1+r}\right)^T}}_{\text{MPC}} \left[\sum_{j=0}^{T} \frac{Y_{t+j}}{(1+r)^j}\right]$$

As $T \to \infty$, MPC gets small

If $r \to 0$, then MPC $\to \frac{1}{T+1}$

Friedman's Permanent Income Hypothesis

Friedman (1957): consumption should equal "permanent income"

Formally, permanent income = annuity value of lifetime wealth

- "annuity value" = constant value of income that equals the present discounted value of anticipated lifetime income

Formally:

$$Y = Y^P + Y^{TR}$$
$$C = Y^P$$

MPC out of permanent income ought to be 1, MPC out of transitory income ought to be 0

Consumption Under Uncertainty

Go back to two periods. But allow future income to be uncertain, where $Y^h_{t+1} > Y^l_{t+1}$:

$$Y_{t+1} = \begin{cases} Y^h_{t+1}, & \text{with probability } p \\ Y^l_{t+1}, & \text{with probability } 1-p \end{cases}$$

Expected value of income:

$$\mathbb{E}\left(Y_{t+1}\right) = pY^h_{t+1} + (1-p)\,Y^l_{t+1}$$

Budget Constraints

Period t constraint the same

Budget constraint must hold in <u>either</u> future state of the world (i.e., income uncertainty translates into consumption uncertainty):

$$C^h_{t+1} = Y^h_{t+1} + (1 + r_t)S_t$$
$$C^l_{t+1} = Y^l_{t+1} + (1 + r_t)S_t$$

Expected Utility and Euler Equation

Household maximizes expected utility:

$$\begin{aligned} U =& u(C_t) + \beta \mathbb{E} u(C_{t+1}) \\ =& u(C_t) + \beta p u(C^h_{t+1}) + \beta(1-p) u(C^l_{t+1}) \end{aligned}$$

Euler equation holds in expectation:

$$\begin{aligned} u'(C_t) =& \beta(1+r_t)\mathbb{E} u'(C_{t+1}) \\ =& \beta(1+r_t) p u'(C^h_{t+1}) + \beta(1+r_t)(1-p) u'(C^l_{t+1}) \end{aligned}$$

Precautionary Saving

If $f(X)$ is convex, then $f\left(\mathbb{E}(X)\right) < \mathbb{E}\left(f\left(X\right)\right)$ (Jensen's Inequality)

$u'(\cdot)$ is a function. If $u'''(\cdot) > 0$, then $u'(\cdot)$ is convex

This means $\mathbb{E}u'(C_{t+1}) \geq u'(\mathbb{E}(C_{t+1}))$

Increase in uncertainty raises expected marginal utility, makes household want to reduce consumption today.

Precautionary saving. Consumption function is (where *un* represents uncertainty over the future):

$$C_t = C^d(\underset{+}{Y_t}, \mathbb{E}(\underset{+}{Y_{t+1}}), \underset{-}{r_t}, \underset{-}{un}).$$

Random Walk Hypothesis

Allow for the future to be uncertain. Assume $\beta(1+r) = 1$:

$$u'(C_t) = \mathbb{E}\left(u'(C_{t+1})\right)$$

But now assume $u'''(\cdot) = 0$ (i.e., quadratic utility). $\Rightarrow$

$$\mathbb{E}\left(C_{t+1}\right) = C_t$$

Rational expectations (Muth 1961, Lucas 1970): expectations are correct <u>on average</u> and forecast errors are <u>unpredictable</u> $\Rightarrow$

$$C_{t+1} - C_t = \varepsilon_{t+1}$$

Where ε_{t+1} is (i) mean zero and (ii) uncorrelated with anything known at date t or earlier

Implications of Random Walk Hypothesis

Hall (1978): changes in consumption should not be predictable

- A predictable change in income (i.e., going from t to $t+1$) ought to be incorporated into consumption as soon as the change in income becomes predictable (in period t)
- Households want to smooth consumption in expectation
- Income will have unforecastable movements, which will lead to unexpected changes in consumption
- But consumption should not react to changes in income that were predictable in the past

Lots of tests of this, generally fails (e.g., retirement, Social Security withholding, monthly paychecks and economic activity)