

Problem Set 4

ECON 30020, Intermediate Macroeconomics, Spring 2026
The University of Notre Dame
Professor Sims

Instructions: Please prepare legible, complete solutions to the following problems. You may work with others, and consult whatever resources you wish, but you are responsible for your own work and must turn in your own assignment. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers and/or figures from Excel – you need not turn in Excel code. An image/scan of the problem set should be uploaded onto the Canvas site via the “assignments” tab no later than 5:00 pm on the due date of March 31.

1. Consider the Neoclassical Model presented in class. The principal equations of the model are:

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$

$$N_t = N^s(w_t, \theta_t)$$

$$N_t = N^d(w_t, A_t, K_t)$$

$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t + G_t$$

$$M_t = P_t M^d(i_t, Y_t)$$

$$r_t = i_t - \pi_{t+1}^e$$

- (a) In words, explain what each equation represents. What are the signs of the partial derivatives? Where relevant, explain what underlying first-order condition motivates each equation.
- (b) What are the exogenous variables? What are the endogenous variables?
- (c) Explain, in words, what it means for Y_{t+1} to be “pseudo-exogenous.”
- (d) Graphically characterize the equilibrium.
- (e) Suppose there is an exogenous increase in A_t . Graphically show how this impacts the equilibrium. For each endogenous variable, indicate whether it increases, decreases, or if the change is ambiguous.
- (f) Re-do the previous problem. But this time assume that the labor supply curve is more elastic with respect to the real wage. For each endogenous variable, indicate whether it responds more, less, the same, or if the answer is ambiguous related to the less elastic labor supply case.
- (g) Go back to the base case. Suppose that there is an exogenous increase in A_{t+1} (anticipated in the present). Graphically show how this impacts the equilibrium. For each endogenous variable, indicate whether it goes up, goes down, is unchanged, or if the effect is ambiguous. If the effect is ambiguous, please indicate why.

2. Suppose we have a simplified model. There is no capital, investment, or government spending. The household is not forward looking. The household's consumption function, labor supply function, and money demand function are given by:

$$C_t = C_0 + mpcY_t - br_t$$

$$N_t = N_0 w_t$$

$$\frac{M_t}{P_t} = -\alpha i_t + \psi Y_t$$

We have $0 < mpc < 1$ (treat this as a fixed parameter) and $b > 0$. C_0 is consumption independent of Y_t and r_t . Assume $C_0 - br_t > 0$ for any value of r_t . N_0 is a constant. The firm has a linear production technology in labor input. Therefore, we have:

$$Y_t = A_t N_t$$

$$w_t = A_t$$

Here, $w_t = A_t$ is the labor demand function, coming from equating the marginal product of labor, A_t , to the wage, w_t . The aggregate resource constraint is that $C_t = Y_t$. Assume expected inflation is $\pi_{t+1}^e = 0$. From the Fisher relationship, this means that $i_t = r_t$.

- (a) Analytically solve for expressions for the equilibrium values of Y_t , r_t , N_t , and P_t . By analytically solve, I mean express these endogenous variables as a function of exogenous variables only.
 - (b) Suppose that there is an exogenous increase in A_t . Use your expressions above to “sign” the effects on these endogenous variables. You don't have to draw the graphs, but explain whether these effects are consistent with what you would get qualitatively in an $IS - Y^s$ diagram.
 - (c) Suppose that there is an exogenous increase in C_0 . Use your expressions above to “sign” the effects on these endogenous variables.
 - (d) Now, suppose that the price level is fixed at $\bar{P}_t$. We replace the labor demand curve with this. This is an algebraic example of the simple sticky price model. Repeat parts (a) - (c) here. How do your answers for the signs of the effects of A_t and C_0 change?
3. Consider a stylized, static representative agent model. A household solves the following problem:

$$\max_{C_t, N_t} U = \ln C_t + \theta \ln(1 - N_t)$$

s.t.

$$C_t = w_t N_t + D_t$$

w_t is the real wage, D_t is the dividend from ownership in the firm. The household takes both as given. A firm produces output according to $Y_t = A_t \bar{K}^\alpha N_t^{1-\alpha}$. Here, $\bar{K}$ is a fixed factor of production that can not be changed. The firm's dividend is $D_t = Y_t - w_t N_t$.

- (a) Solve for the first-order optimality conditions for both the household and the firm. Then impose the market-clearing condition to solve for exact expressions for equilibrium C_t , N_t , and Y_t .
- (b) Now, instead consider a social planner's version of this problem. The social planner's problem features no prices (i.e., no w_t). The social planner simply picks allocations to maximize household utility subject to aggregate scarcity. The social planner's problem is:

$$\max_{C_t, N_t} U = \ln C_t + \theta_t \ln(1 - N_t)$$

s.t.

$$C_t = A_t \bar{K}^\alpha N_t^{1-\alpha}$$

Find the first-order conditions for the planner's problem, and use them to solve for exact values of C_t and N_t .

- (c) What is true about the solutions to the planner's problem and the decentralized competitive equilibrium problem? Can the planner implement any policies to improve upon the competitive equilibrium? Explain briefly.