Lecture 14: Labor Supply, Money Demand, and Equilibrium

ECON 30020: Intermediate Macroeconomics

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Readings

GLS Ch. 12 and 14

Household Problem

$$\max_{C_t, N_t, M_t, C_{t+1}, N_{t+1}} \quad U = u(C_t, 1 - N_t) + v\left(\frac{M_t}{P_t}\right) + \beta u(C_{t+1}, 1 - N_{t+1})$$

s.t.

$$C_t + \frac{C_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t}\frac{M_t}{P_t} = w_t N_t - T_t + D_t + \frac{w_{t+1}N_{t+1} - T_{t+1} + D_{t+1} + D^I_{t+1}}{1 + r_t}$$

Looks nasty, but really just an extension of two-period consumption-saving problem

Optimality Conditions

Solve for one choice variable in terms of others from IBC, plug in, and take derivatives and set equal to zero

1. Euler equation (conceptually the <u>same</u> as before):

$$u_C(C_t, 1 - N_t) = \beta u_C(C_{t+1}, 1 - N_{t+1})(1 + r_t)$$

2. Labor supply condition (<u>static</u> and the same in t and $t+1$):

$$u_L(C_t, 1 - N_t) = w_t u_C(C_t, 1 - N_t)$$

$MB = MC$, equivalently w_t equals MRS between leisure and consumption

3. Money demand condition:

$$v'\left(\frac{M_t}{P_t}\right) = \frac{i_t}{1 + i_t} u_C(C_t, 1 - N_t)$$

$MB = MC$, equivalently $\frac{i_t}{1+i_t}$ equals MRS between money and consumption

From FOC to Decision Rules

Euler equation + IBC $\Rightarrow$ consumption function similar to earlier:

$$C_t = C^d \left(Y_t \underset{+}{-} T_t, Y_{t+1} \underset{+}{-} T_{t+1}, \underset{-}{r_t} \right)$$

Labor supply is trickier: potentially competing income and substitution effects. Assume GHH preferences: no income effect!

$$N_t = N^s(\underset{+}{w_t}, \underset{-}{\theta_t})$$

FOC for money implicitly defines a money demand function:

$$\frac{M_t}{P_t} = M^d(\underset{-}{i_t}, \underset{+}{Y_t})$$

Set of Optimal Decision Rules

Household: consumption, labor supply, money demand:

$$C_t = C^d(Y_t - T_t, Y_{t+1} - T_{t+1}, r_t)$$

$$N_t = N^s(w_t, \theta_t)$$

$$M_t = P_t M^d(i_t, Y_t)$$

Firm: labor demand, investment demand

$$N_t = N^d(w_t, A_t, K_t)$$

$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

Technological Constraint, Law of Motion, and Definition

Production function:

$$Y_t = A_t F(K_t, N_t)$$

Capital accumulation:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

Fisher relationship:

$$i_t = r_t + \pi^e_{t+1}$$

Government

Spending equals revenue:

$$P_t G_t + i_{t-1} P_{t-1} B_{t-1} = P_t T_t + P_t B_t - P_{t-1} B_{t-1} + M_t - M_{t-1}$$

$$P_{t+1} G_{t+1} + i_t P_t B_t = P_{t+1} T_{t+1} + P_{t+1} B_{t+1} - P_t B_t + M_{t+1} - M_t$$

Money is a liability of the government; creating more is like raising more tax revenue

Initial and terminal conditioins: $B_{t-1} = M_{t-1} = 0$, $B_{t+1} = M_{t+1} = 0$

Government Intermporal Budget Constraint

With the initial and terminal conditions, in real terms the government's constraints may be written:

$$G_t = T_t + B_t + \frac{M_t}{P_t}$$

$$G_{t+1} + (1 + i_t)\frac{P_t}{P_{t+1}} B_t + \frac{M_t}{P_{t+1}} = T_{t+1}$$

Use Fisher relationship and combine:

$$G_t + \frac{G_{t+1}}{1 + r_t} = T_t + \frac{T_{t+1}}{1 + r_t} + \frac{i_t}{1 + i_t}\frac{M_t}{P_t}$$

Last term is real "revenue" from "printing" money (seigniorage)

Ricardian Equivalence holds: don't need to know T_t or B_t: plug this into household *IBC* and just left with G_t and G_{t+1}, hence in consumption function treat $T_t = G_t$ and $T_{t+1} = G_{t+1}$

Equilibrium and Resource Constraint

Competitive Equilibrium: set of prices and allocations consistent with everyone behaving optimally and markets clearing

All budget constraints hold with equality (focus on period t only):

$$P_t C_t + P_t G_t + P_t S_t - P_t B_t = P_t Y_t$$

$P_t S_t - P_t B_t$ is aggregate nominal saving ($B_t > 0$ is borrowing)

Market-clearing: saving equals investment (i.e., loanable funds)

Alternatively, just think about this as goods market-clearing implying:

$$Y_t = C_t + I_t + G_t$$

This will look the same in period $t + 1$ (see extra handout)

Prices and Markets

w_t (real): clears the labor market (labor demand equals labor supply)

r_t (real): clears the goods market, $Y_t = C_t + I_t + G_t$ (equivalently, market for loanable funds, $S_t - B_t = I_t$)

i_t (nominal): determined from Fisher relationship, given π^e_{t+1}

P_t: clears the money market (money demand equals money supply)

Variable Types

Exogenous (taken as given, determined "outside" the model:

- A_t, A_{t+1}, G_t, G_{t+1}, M_t, f_t, θ_t, π^e_{t+1}, K_t (initial)

Endogenous (determined "inside" the model):

- Allocations/Quantities: Y_t, C_t, I_t, N_t, K_{t+1} (future)
- Prices: r_t, w_t, P_t, i_t

Ricardian Equivalence: do not need to know T_t, T_{t+1}, or B_t

Future Capital and Future Output

Recall law of motion for capital:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

Treat $K_{t+1} \approx K_t$: over short periods of time, I_t is small relative to capital stock, so doesn't affect it much

This means we can treat Y_{t+1} as pseudo-exogenous:

- Can be affected by changes in period $t + 1$ exogenous variables
- But not affected by changes in period t exogenous variables (i.e., assuming no endogenous propagation through investment)
- Means we don't have to worry about K_{t+1}

Full Set of Equilibrium Conditions

$$C_t = C^d(Y_t - G_t, Y_{t+1} - G_{t+1}, r_t)$$

$$N_t = N^s(w_t, \theta_t)$$

$$M_t = P_t M^d(i_t, Y_t)$$

$$N_t = N^d(w_t, A_t, K_t)$$

$$I_t = I^d(r_t, A_{t+1}, f_t, K_t)$$

$$Y_t = A_t F(K_t, N_t)$$

$$Y_t = C_t + I_t + G_t$$

$$r_t = i_t + \pi^e_{t+1}$$

Eight equations, eight endogenous variables (Y_t, C_t, I_t, N_t, r_t, w_t, P_t, i_t)

- Exogenous: A_t, A_{t+1}, G_t, G_{t+1}, M_t, f_t, θ_t, π^e_{t+1}, K_t