Problem Set 5

ECON 30020, Intermediate Macroeconomics, Spring 2026
The University of Notre Dame
Professor Sims

Instructions: Please prepare legible, complete solutions to the following problems. You may work with others, and consult whatever resources you wish, but you are responsible for your own work and must turn in your own assignment. Please show your work, box or circle final answers, and clearly label any graphs. If the problem set requires work in Excel, you may just report final answers and/or figures from Excel – you need not turn in Excel code. An image/scan of the problem set should be uploaded onto the Canvas site via the "assignments" tab no later than 5:00 pm on the due date of April 28.

1. **Sticky-Wage Aggregate Supply Curve:** Suppose that the nominal wage is predetermined within a period, i.e., $W_t = \bar{W}_t$, where $\bar{W}_t$ is exogenous. With a fixed nominal wage, the household is off its labor supply curve – in essence, at the beginning of a period, the household chooses to post a nominal wage, and commits to working however many hours are demanded at that wage.

   The production function is:

   $$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

   The firm hires labor up until the point at which the real wage, $w_t = \bar{W}_t / P_t$, equals the marginal product of labor. The capital stock, K_t, is fixed within period.

   (a) Derive an exact expression for the aggregate supply curve. Derive an expression for the elasticity of output with respect to the price level, i.e., $d \ln Y_t / d \ln P_t$, and verify that it is positive.

   (b) Which exogenous variables cause the AS to shift, and in which direction? Is this similar or different to the sticky-price AS curve we used earlier?

   (c) In the sticky price model, the real wage is procyclical conditional on an aggregate demand shock. Will that be the case here? Explain briefly (just in words).

2. **Algebra of the IS-MP-AD-PC Model:** Suppose that the IS curve is given by:

   $$Y_t = \bar{A}_t - b r_t$$

   Here, $\bar{A}_t$ is a composite variable that governs anything that would shift the IS curve; $b > 0$ is a parameter. The MP curve is:

   $$r_t = r^* + (\phi - 1)(\pi_t - \pi^*) + ms_t$$

   Here, we assume that $\phi > 1$. r^* and π^* are long-run constants. ms_t is an exogenous monetary policy shock.

The Phillips Curve is given by:

$$\pi_t = \pi_{t-1} + \gamma\left(Y_t - Y_t^f\right) + ps_t$$

$\gamma \geq 0$ is a parameter, ps_t is an exogenous "price shock," and Y_t^f is the hypothetical equilibrium value of output in a neoclassical model with no nominal frictions (we can take this as given with respect to the other endogenous variables presented here). π_{t-1} is the once-lagged value of inflation, taken as given from the perspective of period t. We are making use of adaptive expectations here.

(a) Derive an exact expression for the aggregate demand (AD) curve. What restriction do you need to impose on ϕ to ensure that the AD curve is downward-sloping?

(b) Combine the AD curve with the PC curve to solve for analytical expressions for the equilibrium values of Y_t and π_t as a function of parameters and exogenous variables.

(c) Use your answers from above to compute analytic expressions for the effects of changes in $\bar{A}_t$, Y_t^f, and ms_t, and ps_t on both inflation and output (i.e., compute $dY_t/d\bar{A}_t$ and so on). How does the value of the parameter ϕ impact these effects (i.e., does a bigger ϕ make the impact smaller or bigger?)? From the perspective of the dual mandate, is a larger value of ϕ good or bad conditional on each shock? Explain.

(d) Given adaptive expectations (i.e., $\pi_t^e = \pi_{t-1}$), your solution above essentially amounts to a difference equation (i.e., kind of like the law of motion for capital in the Solow model). Define the steady state as the equilibrium where $\pi_t = \pi_{t-1} = \pi^*$. What must be true of r^* and Y^* then?

(e) Create an Excel file. Assume that $\pi^* = 2$ and $Y^f = 10$. Assume that $\bar{A} = 12$, $b = \gamma = 0.5$, and $\phi = 1.5$. Assume that $ps_t = ms_t = 0$. Create periods running from $1, 2, 3, \ldots 20$. Assuming $\pi_0 = 1$, plot the time paths of π_t and Y_t from periods 1 to 20, assuming nothing else changes. Show your plots. What happens to output and inflation over time?

(f) Re-do the above, but instead assume that $\pi_0 = 2$. What do your plots of output and inflation look like?

(g) Now, suppose that $\pi_0 = 2$. Do the same simulation as in the previous part, but, in period 5, switch $\bar{A}$ from 12 to 13 (it permanently stays there). Produce paths of output and inflation under this change.

(h) Re-do the previous exercise, but with a value of $\phi = 10.5$ (instead of 1.5). Compare and contrast how output and inflation react to the increase in $\bar{A}$ under the two different values of ϕ. From the perspective of the dual mandate, is it better to have a big or small value of ϕ?

3. **The Inflation Target and the ZLB**: Suppose that the MP rule is (abstract from a monetary shock):

$$i_t = i^* + \phi(\pi_t - \pi^*)$$

You may assume that $\phi > 1$. Suppose that r^* is given.

(a) What must be the value of i^*? How do you know. Explain briefly.

(b) Suppose that there is a zero lower bound (ZLB), i.e., $i_t \geq 0$. Find an analytic expression for the value of π_t at which the ZLB binds.

(c) Using the above, find an expression for $\pi_t - \pi^*$ at which the zero lower bound begins to bind.

(d) How does the value of π^* impact how far inflation can fall below target before the ZLB binds? Explain briefly.

(e) How does the value of ϕ impact how far inflation can fall below target before the ZLB binds? Explain briefly. To the extent to which the ZLB is bad, explain why a large value of ϕ may not be too desirable even when the Divine Coincidence holds.

4. **The Expectations Hypothesis of the Term Structure:** Suppose a household lives for <u>four</u> periods, drawing an exogenous endowment of income each period. There is no uncertainty. The household's lifetime utility function is:

$$U = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \beta^3 u(C_{t+3})$$

The flow utility function, $u(\cdot)$, has the usual properties. In period t, the household has the opportunity to save/borrow via a one period bond ($S_{t,t+1}$), a two-period bond, $S_{t,t+2}$, or a three-period bond, $S_{t,t+3}$. Its budget constraint is:

$$C_t + S_{t,t+1} + S_{t,t+2} + S_{t,t+3} = Y_t$$

The one-period bond pays gross interest $1 + i_{t,t+1}$ in $t+1$. The two-period bond pays cumulative gross interest $(1 + i_{t,t+2})^2$ in $t+2$. And the three-period pays cumulative gross interest $(1 + i_{t,t+3})^3$ in $t+3$.

For simplicity, assume that the household cannot change its holdings of the two- or three-period bonds once purchased. In period $t+1$, the household can only borrow or save via another one-period bond, $S_{t+1,t+2}$. Its budget constraint is:

$$C_{t+1} + S_{t+1,t+2} = Y_{t+1} + (1 + i_{t,t+1})S_{t,t+1}$$

In period $t+2$, the household gets (or pays) interest on the two-period bonds it purchased in t. It can again save in a one-period bond. Its budget constraint is:

$$C_{t+2} + S_{t+2,t+3} = Y_{t+2} + (1 + i_{t+1,t+2})S_{t+1,t+2} + (1 + i_{t,t+2})^2 S_{t,t+2}$$

In period $t+3$, the household gets interest plus principle on one-period savings from $t+2$ ($S_{t+2,t+3}$ and cumulative interest plus principle on the three-period bond it bought in t. The budget constraint is:

$$C_{t+3} = Y_{t+3} + (1 + i_{t+2,t+3})S_{t+2,t+3} + (1 + i_{t,t+3})^3 S_{t,t+3}$$

In writing this budget constraint, I have imposed terminal conditions. In all of these terms, the first subscript denotes the date of issuance (e.g., t or $t+1$). The second subscript denotes the date of maturity/payment, e.g., $t+1$ or $t+2$. So, $S_{t,t+2}$ is the amount of a two-period bond purchases in period t that matures in $t+2$; $i_{t,t+2}$ is the interest rate on this bond.

(a) Use calculus to derive the first-order conditions with respect to the choices of $S_{t,t+1}$, $S_{t+1,t+2}$, $S_{t+2,t+3}$, $S_{t,t+2}$, and $S_{t,t+3}$.

(b) Use your optimality conditions to derive relationships between $i_{t,t+2}$ and $i_{t,t+1}$ and $i_{t+1,t+2}$ and $i_{t,t+3}$ and $i_{t,t+1}$, $i_{t+1,t+2}$, and $i_{t+2,t+3}$.