Lecture 5: Asset Pricing
ECON 43370: Financial Crises

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Pricing an Asset

An asset is a claim to future flow benefits: cash flows (financial) or utility flows (real)

How do you price a bunch of items in a grocery cart?

- You sum up the value of every item in the cart

How do you price an asset?

- You sum up the value of every item in the cart

Future Cash Flows

A financial asset (e.g., stock or bond) promises future cash flows

These future cash flows are the items in the cart

An asset might promise an expected cash flow j periods into the future of $\mathbb{E}D_{t+j}$, for $j > 0$

How does one value that cash flow?

Discounting the Future

A dollar received in the future ought to be discounted relative to a dollar received in the present

Why?

- Impatience (subjective discounting)
- Risk (possibility the asset doesn't pay what it promises)

Present Discounted Value

Suppose you discount the future at discount rate, r

The present value of a dollar expected to be received one period in the future is:

$$PV_t = \frac{\mathbb{E}\left[D_{t+1}\right]}{1+r}$$

Equivalently, r is the required return you expect to earn to be willing to own a claim to a dollar in the future (relative to holding a dollar in the present):

$$\mathbb{E}\left[R\right] = \frac{\mathbb{E}[D_{t+1}] - PV_t}{PV_t} = r$$

Present Discounted Value (cont.)

Suppose the cash flow is received two periods from now:

$$PV_t = \frac{\mathbb{E}[D_{t+2}]}{(1+r)^2}$$

For $j > 0$ periods into the future:

$$PV_t = \frac{\mathbb{E}[D_{t+j}]}{(1+r)^j}$$

$(1+r)^j - 1$ is the (compounded) j-period return

The Price of a Generic Asset

You just sum up the value of the cash flows at different points in time:

$$P_t = \frac{\mathbb{E}[D_{t+1}]}{1+r} + \frac{\mathbb{E}[D_{t+2}]}{(1+r)^2} + \frac{\mathbb{E}[D_{t+3}]}{(1+r)^3} + \dots$$

Note, this can be written recursively:

$$P_t = \frac{\mathbb{E}[D_{t+1}] + \mathbb{E}[P_{t+1}]}{1+r}$$

The expected return on holding this asset is:

$$\mathbb{E}[R] = \underbrace{\frac{\mathbb{E}[D_{t+1}]}{P_t}}_{\text{dividend yield}} + \underbrace{\frac{\mathbb{E}[P_{t+1}] - P_t}{P_t}}_{\text{capital gain}}$$

Bonds

A bond is a fixed-income security that promises the holder fixed (and typically pre-specified) cash flows and a face value at the maturity date

- C: coupon payment
- FV: face value
- m: maturity date
- r: required return / discount rate / yield to maturity

The yield to maturity (or just yield) is the discount rate that equates the price of the bond to present discounted value of cash flows

Represents the return from holding the bond to maturity assuming no default

- Realized return may differ from yield if sold prior to maturity or if there is default

Pricing Bonds

For simplicity, assume no uncertainty over cash flows (e.g., government bond)

$$P_t = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{FV}{(1+r)^m}$$

Special cases:

- Discount bond ($C = 0$): $P_t = \frac{FV}{(1+r)^m}$
- Perpetuity / consol ($m \to \infty$): $P_t = \sum_{j=1}^{\infty} \frac{C}{(1+r)^j} = \frac{C}{r}$

Prices, Yields, and Interest Rate Risk

Yield to maturity is just another way to express the price of a bond

Price and yield move opposite each other

Interest rate risk: when required returns rise (i.e., short-term rates go up), bond prices fall

- Logic: if you hold a bond with yield of 5 percent, if prevailing interest rates go up, the only way to sell your bond is at a higher yield (so lower price)
- This means you suffer a capital loss if you are "long" bonds and interest rates rise

Stocks

A stock is a share of ownership (equity)

The periodic cash flows (dividends) from owning stock are not pre-specified

There is no maturity date (and no meaningful face/par value)

$$P_t = \frac{\mathbb{E}[D_{t+1}]}{1+r} + \frac{\mathbb{E}[D_{t+2}]}{(1+r)^2} + \frac{\mathbb{E}[D_{t+3}]}{(1+r)^3} + \dots$$

Or:

$$P_t = \frac{\mathbb{E}[D_{t+1}] + \mathbb{E}[P_{t+1}]}{1+r}$$

Bubbles

One is tempted to express the price of the stock as the PDV of dividends:

$$P_t = \sum_{j=1}^{\infty} \frac{\mathbb{E}[D_{t+j}]}{(1+r)^j}$$

This is implicitly imposing a "no-bubble" condition, $B_t = 0$:

$$P_t = \underbrace{\sum_{j=1}^{\infty} \frac{\mathbb{E}[D_{t+j}]}{(1+r)^j}}_{\text{fundamental price}} + \underbrace{B_t}_{\text{bubble}}$$

Restriction on Bubbles

If a bubble exists, it must be expected to grow:

$$\mathbb{E}[B_{t+1}] = (1 + r)B_t$$

You will "overpay" for an asset if someone in the future will "overpay" by enough so that you earn your required return

We should not observe bubbles for assets with finite lives – no one should overpay in the last period, which unravels the bubble

Gordon Growth Model

Assume dividends grow at a deterministic, constant rate, g:

$$D_{t+j} = (1+g)^j D_t$$

Imposing a no bubble condition, we get:

$$P_t = \frac{D_{t+1}}{r-g}$$

Stock prices are higher the higher is g or lower is required return, r

Why Care About Asset Prices?

Asset prices convey information about expectations about the future

Asset prices are relevant for expenditure / aggregate demand:

- Consumption: wealth effects and easing of borrowing constraints
- Investment: reduces cost of capital (i.e., cheaper to issue new shares to fund investment)

Balance sheets and financial accelerator: falling asset prices weaken balance sheets and can trigger runs

- Financial crises are almost always preceded by some kind of "bubble bursting"

Discount Rate

This is all well and good, but what discount rate do we use to price future cash flows?

Discount rate is the <u>required</u> return to hold a claim on future cash flows

Basic idea: the <u>riskier</u> the future cash flows, the higher required return. Use a larger discount rate

Risk classifications: stock $>$ corporate bonds $>$ long-term government bonds $>$ short-term government bonds

Monetary policy 101: monetary policy influences short-term, riskless interest rate. Everything else is "priced off" of that (risk and term premia)

Micro-Founded Asset Pricing

Household lives for two periods, earns exogenous income stream

Can buy (save) or sell (borrow) some asset, A, at price P

Asset offers some cash flow in the future, D_{t+1} (may be stochastic)

Can also save (or borrow) through a risk-less asset at interest rate r_t

Household Problem: Deterministic Case

$$\max \quad U = u(C_t) + \beta u(C_{t+1}), \quad 0 < \beta < 1$$

s.t.

$$C_t + P_t A_t + S_t = Y_t$$

$$C_{t+1} = Y_{t+1} + D_{t+1} A_t + (1 + r_t) S_t$$

Euler Equations

Optimality conditions:

$$1 = \frac{\beta u'(C_{t+1})}{u'(C_t)}(1 + r_t)$$

$$P_t = \frac{\beta u'(C_{t+1})}{u'(C_t)} D_{t+1}$$

$\frac{\beta u'(C_{t+1})}{u'(C_t)}$ is the marginal rate of substitution (MRS) between future and current consumption: the <u>stochastic discount factor</u>

If no uncertainty over D_{t+1}, you use the same discount rate to price both assets:

$$P_t = \frac{D_{t+1}}{1 + r_t}$$

Uncertainty

If there is uncertainty about the future (in income, in asset payouts, or both) similar expressions hold, but in expectation:

$$1 = \mathbb{E}\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}(1 + r_t)\right]$$

$$P_t = \mathbb{E}\left[\frac{\beta u'(C_{t+1})}{u'(C_t)} D_{t+1}\right]$$

This is why we call the MRS between future and current consumption the <u>stochastic</u> discount factor: with uncertainty, from the perspective of period t, we don't know $u'(C_{t+1})$

The Risk-free Rate

If r_t is risk-free, we can write:

$$\frac{1}{1+r_t} = \mathbb{E}\left[\frac{\beta u'(C_{t+1})}{u'(C_t)}\right]$$

But this does <u>not</u> imply that we can write the price of the other asset as:

$$P_t = \frac{\mathbb{E}[D_{t+1}]}{1+r_t}$$

Why? Expected value of the product of two random variables does <u>not</u> equal the product of the expected values

Example

Suppose there are four possible states of the world in the future, where $Y^h_{t+1} \geq Y^l_{t+1}$ and $D^h_{t+1} \geq D^l_{t+1}$

1. Y^h_{t+1} and D^h_{t+1} (good, good). Probability p_1
2. Y^h_{t+1} and D^l_{t+1} (good, bad). Probability p_2
3. Y^l_{t+1} and D^h_{t+1} (bad, good). Probability p_3
4. Y^l_{t+1} and D^l_{t+1} (bad, bad). Probability $p_4 = 1 - p_1 - p_2 - p_3$

Budget constraint must hold in all future states of the world ($j = 1, \ldots, 4$):

$$C_{t+1}(j) = Y_{t+1}(j) + D_{t+1}(j)A_t + (1 + r_t)S_t$$

Values

Assume $\beta = 0.95$ and $Y_t = 2$

Assume supplies of both assets are fixed at $S_t = 0$ and $A_t = 1$

Assume $Y_{t+1}^h = 1.7$ and $Y_{t+1}^l = 1.4$

Assume $D_{t+1}^h = 0.4$ and $D_{t+1}^l = 0.2$

Assume $p_1 = 0.5$ and $p_4 = 0.5$

Utility function is the natural log: $u(C_t) = \ln C_t$

Can numerically solve for r_t and P_t, as well as the required return on the stock, r_t^s

Numeric Results

With these values, I get $r_t = 0.099$, $P_t = 0.261$

The required return (discount rate) on the stock satisfies:

$$P_t = \frac{\mathbb{E}[D_{t+1}]}{1 + r_t^s}$$

I get $r_t^s = 0.151$

The stock offers about a 5 percentage point higher return to compensate for <u>risk</u>

Covariance, Not Variance

But what do we mean by risk, exactly?

What matters, economically, is how the payout from the stock co-varies with how you value this payout

As specified, the stock has a high dividend when consumption is high (marginal utility of consumption is low), and vice-versa

A household dislikes this because it makes it harder to smooth consumption: demands a higher return

Suppose I make $p_2 = p_3 = 0.5$. Then the required return on the stock is $r_t^s = 0.114$ while the risk-free return is $r_t = 0.124$. Stock trades at a lower required return because now the stock pays you more when you most value it

- An asset with this kind of payout structure is like a hedge / insurance, so you accept a low (or potentially even negative) return

Returns Across Asset Classes

Expected returns = discount rate used to evaluate future cash flows

Stocks are the riskiest of standard financial securities:

- Residual claimant on profits (junior relative to debt)
- Cash flows (dividends) are uncertain and likely to be <u>procyclical</u>

Corporate bonds are the next riskiest: default rates are likely countercyclical, so cash flows are procyclical

Long-term government bonds: no (or nearly no) default risk, but subject to interest rate risk

Short-term government bonds: essentially no risk, money-like