Lecture 4: Diamond-Dybvig and Bank Runs

ECON 43370: Financial Crises

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Suggested Reading

"Banks and Liquidity Creation: A Simple Exposition of the Diamond-Dybvig Model" Douglas Diamond, *Federal Reserve Bank of Richmond Economic Quarterly*, Spring 2007. Link

Model Basics

Time lasts for three periods, $T = 0$ (present), $T = 1$, and $T = 2$

There are many households (price-takers), each endowed with one dollar

The households have no need to consume in present. They will need to consume in the future, but they don't know when

Probability $0 \leq t \leq 1$ that a household will be "early" (has to consume in $T = 1$)

Probability $1 - t$ that household is "late" and can wait to consume until $T = 2$

No aggregate uncertainty

Direct Investment Opportunity

The household has the option of storage (holding cash). One dollar saved today generates one dollar available in either period in the future

There exists an investment opportunity (asset) without any uncertainty. It is indivisible

One dollar invested in $T = 0$ generates r_1 (gross) if withdrawn in $T = 1$ and $r_2 \geq r_1$ if held until $T = 2$

$\frac{r_1}{r_2}$ is a measure of the liquidity of the asset

The further from 1 this ratio is, the less liquid is the asset (i.e., the bigger penalty one incurs for early withdrawal)

Returns on investment don't depend on how much is invested

Household Utility

Household receives a concave utility flow from consumption (e.g., $u'(\cdot) > 0$ and $u''(\cdot) < 0$)

No discounting of the future

Expected utility from storage:

$$\mathbb{E}\left[U^S\right] = tu(1) + (1-t)u(1) = u(1)$$

Expected utility from investment:

$$\mathbb{E}\left[U^I\right] = tu(r_1) + (1-t)u(r_2)$$

Household decision: invest if $\mathbb{E}\left[U^I\right] \geq \mathbb{E}\left[U^S\right]$

Example

Suppose $r_1 = 1/2$, $r_2 = 3/2$, and $t = 2/5$

The expected return from investing is positive:

$$\mathbb{E}\left[r\right] = tr_1 + (1-t)r_2 = \frac{2}{5} \times \frac{1}{2} + \frac{3}{5} \times \frac{3}{2} = \frac{11}{10} = 1.1$$

Suppose the utility function is $U(C) = 1 - 1/C$. Expected utility from storage is zero. Expected utility from direct investment is:

$$\mathbb{E}\left[U^I\right] = \frac{2}{5}\left(1 - 1/r_1\right) + \frac{3}{5}\left(1 - 1/r_2\right)$$

$$\mathbb{E}\left[U^I\right] = \frac{2}{5}\left(1 - 2\right) + \frac{3}{5}\left(1 - \frac{2}{3}\right) = -\frac{1}{5}$$

The household will <u>not</u> directly invest!

Enter a Bank

Suppose we have a "bank": it takes deposits from households, stores some, invests some

Suppose there are 1000 households. Exactly fraction t will need to consume early

By aggregating deposits, bank can exploit the lack of aggregate uncertainty and offer the households something better than direct investment or storage – indirect investment

In the process, the bank can create liquidity

Specifics

Bank takes 1000 deposits in $T = 0$

Offers depositors an asset that pays r_1^d if withdrawn in $T = 1$ and r_2^d if held until $T = 2$

Bank anticipates $1000t$ withdrawals in $T = 1$. Stores $S = r_1^d 1000t$ to meet this, invests the rest

Has $1000 - S = 1000 - r_1^d 1000t$ leftover to pay out to late depositors, so:

$$r_2^d 1000(1 - t) = r_2 \left(1000 - r_1^d 1000t\right)$$

$$r_2^d = \frac{r_2}{1 - t}(1 - r_1^d t)$$

Optimal Liquidity

$$\max_{r_1^d, r_2^d} \quad \mathbb{E}\left[U^d\right] = tu\left(r_1^d\right) + (1-t)u\left(r_2^d\right)$$

s.t.

$$r_2^d = \frac{r_2}{1-t}(1 - r_1^d t)$$

FOC:

$$tu'(r_1^d) = (1-t)u'(r_2^d)\frac{t}{1-t}r_2$$

Or:

$$u'(r_1^d) = u'(r_2^d)r_2$$

Example

With the utility function $U(C) = 1 - 1/C$, this implies:

$$r_1^d = r_2^{-1/2} r_2^d$$

With $r_2 = 3/2$, this means $r_1^d = 1.124$ and $r_2^d = 1.376$

This is more liquid than the underlying investment (i.e., $r_1^d / r_2^d = 0.82 > r_1 / r_2$

A household prefers this to storage, $\mathbb{E}\left[U^D\right] = 0.2081 > 0$

Alternative Example

In previous example, household wouldn't choose to invest directly

What if, instead, $r_1 = 5/4$ and $r_2 = 2$? Assume same utility function and $t = 2/5$

$$\mathbb{E}\left[U^I\right] = \frac{2}{5}\left(1 - \frac{4}{5}\right) + \frac{3}{5}\left(1 - \frac{1}{2}\right) = \frac{19}{50}$$

This is greater than the expected utility from storage. Household would choose to invest directly

Is there still a role for a bank?

Yes!

The bank takes 1000 deposits. It promises early withdrawals r_1^d and late r_2^d

Difference: bank doesn't store anything. It invests everything and liquidates L to meet withdrawal demands in $T = 1$:

$$r_1^d 1000t = r_1 L$$

$$r_2^d 1000(1 - t) = r_2(1000 - L)$$

So:

$$r_2^d = \frac{r_2}{1 - t}\left(1 - \frac{r_1^d}{r_1}t\right)$$

Optimal Liquidity

$$\max_{r_1^d, r_2^d} \quad \mathbb{E}\left[U^d\right] = tu\left(r_1^d\right) + (1-t)u\left(r_2^d\right)$$

s.t.

$$r_2^d = \frac{r_2}{1-t}\left(1 - \frac{r_1^d}{r_1}t\right)$$

FOC:

$$tu'\left(r_1^d\right) = (1-t)u'\left(r_2^d\right)\frac{r_2}{r_1}\frac{t}{1-t}$$

$$u'\left(r_1^d\right) = u'\left(r_2^d\right)\frac{r_2}{r_1}$$

Example

With the utility function $u(C) = 1 - 1/C$, this implies:

$$r_1^d = \left(\frac{r_1}{r_2}\right)^{1/2} r_2^d$$

Implies $r_2^d = 1.8084$ and $r_1^d = 1.4296$. Expected utility is 0.3884, higher than expected utility from direct investment

Bank has created liquidity: $r_1^d / r_2^d = 0.791 > r_1 / r_2 = 0.625$

Runs

This is all well and good

But what if more people than expected want their money out in $T = 1$?

- Bank would have to liquidate some of its investments at a loss
- To meet early withdrawal demands at the promised r_1^d, would have to reduce the r_2^d it can pay to $\widehat{r}_2^d < r_2^d$
- If enough additional early withdrawals, could have $\widehat{r}_2^d < r_1^d$.
- If that's the case, everyone tries to withdraw, and the bank fails

Is Running Irrational?

No. If you think $\widehat{r}_2^d < r_1^d$ – for any reason – the best response is to withdraw

Why might you think this?

- Concerns about asset under-performance
- Concerns that other people are going to withdraw early

Bank Runs

Go with our earlier example. $r_1 = 1/2$, $r_2 = 3/2$, $t = 2/5$

Optimal liquidity provision: $r_1^d = 1.124$ and $r_2^d = 1.376$

Suppose "late" households wake up in $T = 1$ and are endowed with the belief that a percentage, $t + f$, of the population are going to withdraw "early"

- t is the percentage of the population who have to withdraw in $T = 1$
- $f \geq 0$ are "late types" (expressed as a percentage of the overall population) who choose to withdraw early

Runs Lead to Losses

Suppose everyone who withdraws in $T = 1$ gets r_1^d

Bank must liquidate $\frac{r_1^d}{r_1}$ for every additional withdrawal in $T = 1$

- i.e., with $r_1^d = 1.124$ and $r_1 = 1/2$, must sell about 2.25 units of the investment
- This leaves less to be distributed in $T = 2$
- If you expect enough other to withdraw early, could be optimal to also withdraw early

To Run or Not

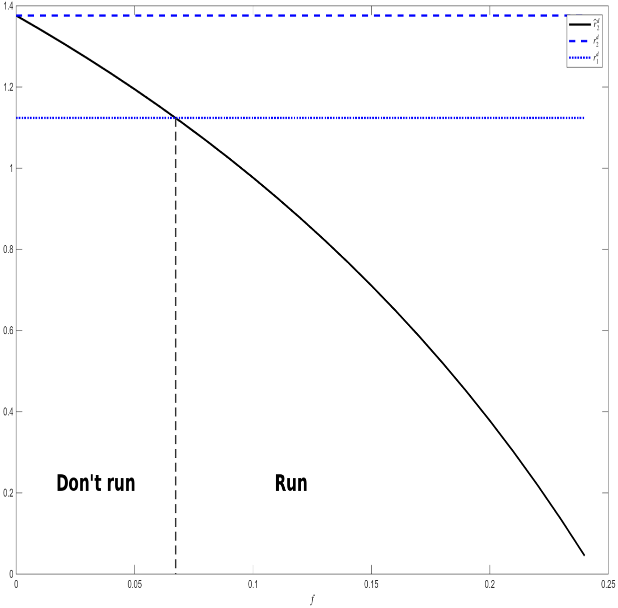
$$\widehat{r}_2^d 1000(1-t-f) = r_2 \left(1000 - r_1^d 1000t - \frac{r_1^d}{r_1} 1000f\right)$$

$$\rightarrow$$

$$\widehat{r}_2^d = \frac{r_2}{1-t-f}\left(1 - r_1^d \left(t + \frac{f}{r_1}\right)\right)$$

- $f = 0$: $\widehat{r}_2^d = r_2^d > r_1^d$ (don't withdraw)
- $f = 0.05$: $\widehat{r}_2^d = 1.195 > r_1^d$ (don't withdraw)
- $f = 0.10$: $\widehat{r}_2^d = 0.9768 < r_1^d$ (withdraw!)

To Run or Not



Multiple Equilibria

There are two (rational) equilibria here depending on the initial belief about f (equilibrium: everyone playing according to their best-response function and has no incentive to deviate)

- $f \leq f^c$ ($f^c \approx 0.07$): exactly fraction t withdraws in $T = 1$ (the good equilibrium)
- $f > f^c$: everyone withdraws early (the bad equilibrium)

"First come, first served" feature of debt:

- If everyone withdraws early, the maximum amount of cash the bank can come up with is about $725
- At $r_1^d = 1.124$, only 645 depositors can get the promised r_1^d in $T = 1$ before the bank is literally out of money
- Heightens incentives to run if $f > 0$

Liquidity Transformation: Costs and Benefits

Liquidity/maturity transformation is highly valuable: savers get <u>indirect</u> access to high-return projects but get liquidity they desire

But this feature has a bug: system is inherently susceptible to runs

This “run dynamic” is at the heart of financial crises and is potentially quite costly

How to Prevent and/or Mitigate Runs

1. High reserve / liquidity requirements
2. Deposit insurance
3. Suspension of convertibility (e.g., "bank holiday") with information provision
4. Lender of last resort

Debt vs. Equity

Runs are possible because of debt contracts (particularly short-term debt) like deposits

Runs can't happen with equity or 100-percent reserve requirements

- Equity-financed banking
- Narrow bank