25TH GRADUATE STUDENT CONFERENCE IN LOGIC

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Abstracts

Plenary Address

An excursion into combinatorics guided by a model theorist

Aristomenis D Papadopoulos
University of Maryland

Counting things is great fun. It's even greater fun when said things are easy to count. Unfortunately, figuring out if your things are easy to count before you start counting them can be a real hassle sometimes. By what some may call a funny little accident, it turns out that model-theoretic language can often tell us if things are easy to count or not.

What a mysterious introduction, I can hear you thinking... He's mentioned "counting things" like six times and yet he's not really said anything. In my talk, I promise I'll say something. At the very least, I'll do my best to discuss Szemerédi regularity, Erdös-Hajnal, and the Zarankiewicz problem, in various model-theoretically tame contexts.

Internality and binding groups

David Meretzky
University of Notre Dame

Morley's conclusion to Łos' celebrated conjecture about possible modes of categoricity introduced a number of methods into model theory, blossoming as Shelah-style classification theory. Zilber's further analysis of the fine structure of totally categorical theories introduced another paradigm into model theory, that of geometric stability theory. This talk will describe the key notion of internality and Zilber's binding group in geometric stability theory. Time permitting, I will discuss some classic results of Zilber on non-finite axiomatizability of totally categorical theories.

Hazal Sena Aydogdu, *NIP formulas and generic sets in pseudofinite groups.*
University of Illinois Chicago

We introduce the notions of NIP and VC-dimension, and explore their relationship. Using a theorem about VC-dimension, one can derive an analogous result in the setting of pseudofinite structures under the NIP assumption. As a corollary of this in the group context, we obtain a qualitative result for the genericity of formulas with positive pseudofinite counting measure.

Rishi Banerjee, *Baire Category and the Lopez-Escobar Theorem.*
University of Illinois, Chicago

This will be an expository talk on the Lopez-Escobar theorem, a fundamental result in the model theory of infinitary logic stating that the syntactic complexity of infinitary sentences mirrors the topological complexity of the classes they axiomatize. More specifically, Lopez-Escobar showed that the Lw1w-elementary classes are exactly the isomorphism-invariant Borel subsets of Mod(L). Vaught later improved on this result by showing that it holds "level-by-level" through the Borel hierarchy. Vaught's proof also introduced powerful new Baire-categorical techniques to the study of Lw1w through the use of the Vaught transform. We will discuss some of the central concepts in Vaught's proof and their connection to computability-theoretic forcing.

Sapir Ben-Shahar, *Almost computable presentations of Polish spaces and Polish groups.*
University of Wisconsin-Madison

From the very beginning of computability theory, questions about the computability of real numbers, real-valued functions, metric spaces, Banach spaces, topological groups, and so on, have been of great interest to computability theorists. In fact, Turing's 1936 paper introducing Turing machines was primarily focused on computable real numbers. Since then, a number of notions for computably presenting or computably approximating complete separable (i.e. Polish) spaces have arisen independently. Some of these have only been separated or shown to be equivalent fairly recently. I will introduce a few of the most well-studied computability notions in this area and talk about a surprising result showing the existence of Polish spaces and Polish groups that are very close to being computable but are actually not.

Gabriel Day, *New Results on Colored Tree Properties.*
University of Notre Dame

In model theory, tree properties such as TP, TP1 and TP2 have been important to the classification of first-order theories. My research examines variations of these properties in which the indexing trees are enriched with additional structure, in particular, predicates for two or more colors. In this talk, I present recent advances in this research, including characterizations of well-known model-theoretic properties in terms of colored trees.

Gavin Dooley, *Iterated jumps of weak König's lemma.*
University of Notre Dame

In reverse mathematics, there is a sense in which the cohesive principle (COH) is the jump of weak König's lemma (WKL). We explain this idea, and then prove that all the finitely iterated jumps of WKL are pairwise logically independent from each other over ω-models of RCA_0. Along the way, and more importantly, we elucidate a general technique for separating principles over ω-models of RCA_0.

Yutong Duan, *Algebraic independence of solutions of the Lotka-Volterra system.*
University of Illinois, Chicago

Lotka-Volterra system of equations describes the dynamics of biological systems in which two species interact. Formally, the equations are a parameterized family of planar polynomial vector fields. By regarding its solution sets as a definable set in a monster model of the theory of differentially closed

fields of characteristic zero, it is strongly minimal, geometrically trivial and strictly disintegrated (that is, the generic solutions are algebraic independent).

Daniel Gonzalez, *How to Decide the Continuum Hypothesis.*
University of California, Berkeley

In this talk, I will discuss a semi-philosophical paper by John Steel on Generically Invariant Set Theory. We will discuss the language, axioms, and philosophical motivations, and philosophical theses that naturally arise when considering the relationship between the language of Generically Invariant Set Theory and standard first order set theory. We will look at what these theses imply about whether the continuum hypothesis expresses a mathematical proposition, and if not, whether the standard language of set theory can be clarified so that it does. For the last issue, we will consider a philosophical argument for adopting the axiom $V =$ ultimate-L into our foundational theory of mathematics.

Logan Heath, *Degree Spectra for Classification Theory .*
University of Wisconsin-Madison

The spectrum, Spec(T), of a countable, complete theory T, is the collection of Turing degrees of countable models of T. This notion of spectrum allows us a way of using computability theoretic tools to divide between distinct classes of first order theories by answering questions of the form "Is there a theory T, with model theoretic property X, such that no theory with the same spectrum has model theoretic property Y?" We will give a brief summary of what results are known in this area and present (the negative part) of one such result: The PA degrees are the spectrum of a superstable theory T, but not the spectrum of any ω-stable theory.

Bilge Koksal, *Lascar Strong Equivalence Relations.*
Cornell University

I will introduce the Lascar strong equivalence relation (LSER) associated to a complete first order theory. This induces a sigma-compact equivalence relation on a compact totally disconnected Polish space. We introduce a new way of classifying such equivalence relations so that the induced LSER is an invariant of the associated theory. I will discuss some questions about this classification and an inverse Galois problem.

Dhruv Kulshreshtha, *The Well-foundedness of Cardinals and the Axiom of Choice.*
University of Michigan

The question of whether the well-foundedness of cardinals implies the axiom of choice has been raised on many occasions, and remains a major open problem. In the complete absence of choice, however, there turn out to be various notions of well-foundedness. These notions will be discussed, alongside the implications among them. Some interactions between these and other consequences of choice will also be addressed. For example, assuming the Partition Principle, all these well-foundedness principles are provably equivalent. Furthermore, one of these principles, concerning surjections, implies the Dual Cantor-Schröder-Bernstein theorem. This talk is based on joint work with Andreas Blass.

Joey Lakerdas-Gayle, *Isomorphism Spectra and Computably Composite Structures.*
University of Waterloo

The *isomorphism spectrum* of a pair of computable structures $\mathcal{A}$ and $\mathcal{B}$ is the set of Turing degrees that compute an isomorphism from $\mathcal{A}$ to $\mathcal{B}$. We show that the class of isomorphism spectra is closed under computable unions and use this to construct isomorphism spectra that are not finitely generated.

Xiang Li, *Interpreting First-Order Structures in Graphs.*
The Ohio State University

In this talk, we explore interpretations and bi-interpretations within first-order logic. We prove the existence of a bi-interpretation between the class of structures of finite relational languages and the class of models of graph theories. We identify an oversight in Wilfrid Hodges' Model Theory and present a new construction of the interpreting graph that ensures the bi-interpretation holds while guaranteeing that the interpreted graph satisfies niceness property in Mekler's Construction.

Brian Ransome, *On BPI in Symmetric Extensions.*
University of California, Irvine

We give a new proof that the Boolean Prime Ideal theorem holds in the first Cohen model, based on Harrington's insightful proof of a dense-set version of the Halpern-Läuchli theorem. The Halpern-Läuchli theorem and its many variants are known to be closely connected to the independence of the axiom of choice from BPI. We discuss the relationship between these two theorems that is implicit in our work. We also show that the methods given in our proof serve to reprove and unify other results in the literature surrounding BPI in symmetric extensions, as well as prove several new results of interest.

Karthik Ravishankar, *Characterizing Ahmad pairs in the Σ^0_2 e-degrees.*
University of Wisconsin-Madison

Sets (A, B) form an Ahmad pair if every degree strictly below A is also below B while A itself is not below B. Recently this phenomenon has attracted a lot of attention as it plays a role in answering the AE theory of the Σ_2 e-degrees. In this talk I show that the left half of an Ahmad pair must be low_3 and join irreducible. We also show that the right half cannot be low_3 thereby giving a separation between A and B. It follows that there cannot be an Ahmad triple i.e. sets A, B, C such that (A, B) and (B, C) are both Ahmad pairs. This is joint work with Joseph Miller and Mariya Soskova.

John Spoerl, *A Funny Thing About (Bounded) Ultraproducts.*
University of Wisconsin-Madison

Taking an ultraproduct produces a nice structure, right? Well, sort of. We'll discuss a modification of the ultraproduct construction that can fail to preserve first-order properties, but saves some important second-order properties. Even stranger, any $\forall\exists$ property which is preserved by this construction is more simple than it appears.

Atticus StoneStrom, *On f-generic types in NIP groups.*
University of Notre Dame

In stable group theory, a key role is played by "generic" types. In the more general setting of definably amenable NIP groups, these no longer suffice, and the appropriate substitute is given by

"f-generic" types. I will present a positive answer to a question of Chernikov and Simon, showing that, for an NIP group, definable amenability is equivalent to the existence of f-generic types. As a quick application, I will show any dp-minimal group is definably amenable.

Hongyu Zhu, *Seeing the forest does not account for the trees.*
University of Wisconsin-Madison

Say a first-order theory (or a type) has bounded axiomatization if it has an axiomatization by $\forall_n$-formulas for some finite n. In this talk, we will discuss basic properties of theories and types with (or without) bounded axiomatizations, and in particular whether boundedness of theories implies that of types. (The meaning of the title will be explained in due time.)

Tan Özalp, *Initial Tukey Structure Below a Stable Ordered-Union Ultrafilter.*
University of Notre Dame

Tukey introduced the notion of Tukey ordering for partial orders to study convergence in general topological spaces. The Tukey ordering of a particular class of partial orders, namely the class of ultrafilters has been a major topic of interest in the last 16+ years. Since Todorcevic's (2012) proof that Ramsey ultrafilters are Tukey minimal, the investigation of initial Tukey structures below special ultrafilters has been an important part of this work. Subsequently, the initial Tukey structures below ultrafilters forced by Laflamme's partial orders were classified by Dobrinen and Todorcevic (2014 and 2015), below ultrafilters forced by $\mathcal{P}(\omega^k)/\mathrm{Fin}^{\otimes k}$ for all $k \geq 2$ were classified by Dobrinen (2016), and below ultrafilters associated to topological Ramsey spaces constructed from Fraïssé classes were classified by Dobrinen, Mijares and Trujillo (2017). The classification of the initial Tukey structure below a stable ordered-union ultrafilter was a question of the 2011 paper of Dobrinen and Todorcevic, which also appeared in Kuzeljevic and Raghavan's survey paper (2024). Continuing this line of work, we prove that there are exactly 5 Tukey classes below a stable ordered-union ultrafilter. For this, we first simplify a canonization theorem of Klein and Spinas (2005) for Borel functions on the Milliken space of infinite block sequences to the context of fronts on this space, and then we utilize this in the proof of the initial Tukey structure result.