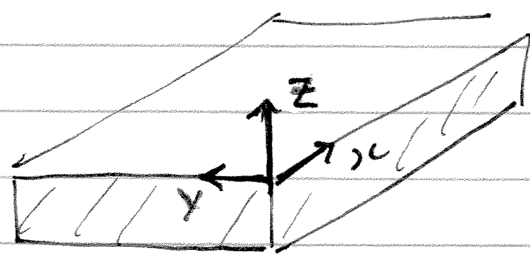
Optics & Spectroscopy Lecture 7

~ EM waves at metal-dielectric interfaces

System:



~ assume wave travels in the x-direction

$$\vec{E} = \vec{E}_0 e^{i(kx-\omega t)}$$

Maxwell's Equations
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

where $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ $\leftarrow$ $\vec{P} = \epsilon_0 \chi \vec{E}$

$\vec{B} = \mu_0 \vec{H}$

$$\Rightarrow \vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

and the dielectric constant (rel. permittivity) is defined as: $\epsilon_r = (1+\chi)$

For "harmonic" waves: $\dfrac{\partial \vec{E}}{\partial t} = -i\omega \vec{E}$

$\quad$ & $\dfrac{\partial \vec{H}}{\partial t} = -i\omega \vec{H}$

$\Rightarrow \nabla \times \vec{E} = i\omega \mu_0 \vec{H} \qquad - (1)$

& $\nabla \times \vec{H} = -i\omega \epsilon_0 \epsilon_r \vec{E} \qquad - (2)$

From (1): $\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$

x-component: $\dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial x} = i\omega \mu_0 H_x$

y-component: $-\left(\dfrac{\partial E_z}{\partial x} - \dfrac{\partial E_x}{\partial z}\right) = i\omega \mu_0 H_y$

z-component: $\dfrac{\partial E_y}{\partial x} - \dfrac{\partial E_x}{\partial y} = i\omega \mu_0 H_z$

From (2):

x-component: $\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -i\omega_0 \epsilon_0 \epsilon_r E_x$

y-component: $-\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) = -i\omega \epsilon_0 \epsilon_r E_y$

z-component: $\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\omega \epsilon_0 \epsilon_r E_z$

Note: wave is "homogeneous" in the y-direction + propagates along x

$\Rightarrow \frac{\partial E}{\partial y} = 0 \quad + \quad \frac{\partial E}{\partial x} = ikE$

$\Rightarrow$ Equations for H_i become:

$$-\frac{\partial E_y}{\partial z} = i\omega\mu_0 H_x$$

$$-ikE_z + \frac{\partial E_x}{\partial z} = i\omega\mu_0 H_y$$

$$ikE_y = i\omega\mu_0 H_z$$

Likewise, equations for E_i become:

$$-\frac{\partial H_x}{\partial z} = -i\omega\varepsilon_0\varepsilon_r E_x$$

$$-ikH_z + \frac{\partial H_x}{\partial z} = -i\omega\varepsilon_0\varepsilon_r E_y$$

$$ikH_y = -i\omega\varepsilon_0\varepsilon_r E_z$$

Two types of modes to consider:

TM = "transverse magnetic"

TE = "transverse electric"

~ look @ TM modes first

here $H_y \neq 0 \Rightarrow E_y = 0$

$\Rightarrow H_x = H_z = 0$

i.e. only E_x, E_z, H_y are non-zero

$\Rightarrow$ left w/ three equations:

$$\frac{\partial E_x}{\partial z} - ik E_z = i\omega\mu_0 H_y \qquad - (1)$$

$$\frac{-\partial H_y}{\partial z} = -i\omega \varepsilon_0 \varepsilon_r E_x \qquad - (2)$$

$$ik H_y = -i\omega \varepsilon_0 \varepsilon_r E_z \qquad - (3)$$

~ rewrite (2) + (3)

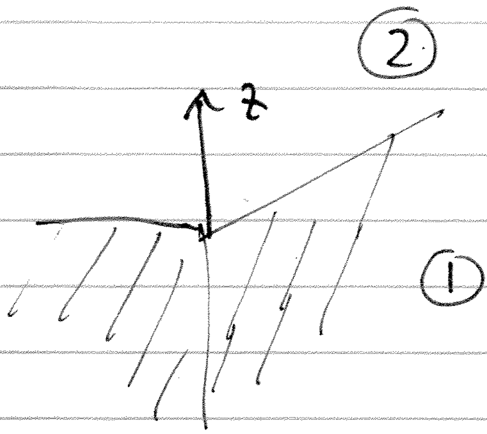
$$E_x = \frac{-i}{\omega \varepsilon_0 \varepsilon_r} \frac{\partial H_y}{\partial z} \; ; \quad E_z = \frac{-k}{\omega \varepsilon_0 \varepsilon_r} H_y$$

~ substitute into (1)

$$\frac{-i}{\omega \varepsilon_0 \varepsilon_r} \frac{\partial^2 H_y}{\partial z^2} + \frac{i k^2}{\omega \varepsilon_0 \varepsilon_r} H_y = i\omega\mu_0 H_y$$

$$\Rightarrow \; -\frac{\partial^2 H_y}{\partial z^2} = \left\{ \omega^2 \mu_0 \varepsilon_0 \varepsilon_r - k^2 \right\} H_y$$

~ rewrite as $$-\frac{\partial^2 H_y}{\partial z^2} = \left\{ \frac{\omega^2 \varepsilon}{c_0^2} - k^2 \right\} H_y$$



medium 2 ~ dielectric

$\Rightarrow$ ε_2 ~ real

medium 1 ~ metal

$\Rightarrow$ ε_1 ~ complex

+ $Re(\varepsilon_1) < 0$

~ interested in waves that are confined to the interface

$\Rightarrow$ decay as: e^{-z/δ_2} $z > 0$

e^{z/δ_1} $z < 0$

$\Rightarrow$ $z > 0$ $H_y(2) = A_2 e^{ikx} e^{-z/\delta_2}$ — (1)

$z < 0$ $H_y(1) = A_1 e^{ikx} e^{z/\delta_1}$ — (2)

From (1):

$$E_x = \frac{-i}{\omega \epsilon_0 \epsilon_r} \frac{\partial H_y}{\partial z} = \frac{-i}{\omega \epsilon_0 \epsilon_2} \left(\frac{-A_2}{\delta_2} \right) e^{ikx} e^{-z/\delta_2}$$

$$E_z = \frac{-k}{\omega \epsilon_0 \epsilon_r} H_y = \frac{-k A_2}{\omega \epsilon_0 \epsilon_2} e^{ikx} e^{-z/\delta_2}$$

Likewise, from (2):

$$E_x = \frac{-i}{\omega \epsilon_0 \epsilon_1} \left(\frac{A_1}{\delta_1} \right) e^{ikx} e^{+z/\delta_1}$$

$$E_z = \frac{-k A_1}{\omega \epsilon_0 \epsilon_1} e^{ikx} e^{+z/\delta_1}$$

~ tangential components of E & H have to be equal at the interface

$\Rightarrow H_y(1) = H_y(2)$ at $z=0$

& $E_x(1) = E_x(2)$ at $z=0$

from b.c. for H_y: $A_1 = A_2$

& from the b.c. for E_x:

$$\frac{i}{\omega \varepsilon_0 \varepsilon_2}\left(\frac{A_2}{\delta_2}\right) = \frac{-i}{\omega \varepsilon_0 \varepsilon_1}\left(\frac{A_1}{\delta_1}\right)$$

$$\Rightarrow \quad \frac{\delta_1}{\delta_2} = -\frac{\varepsilon_2}{\varepsilon_1}$$

② = dielectric $\Rightarrow$ $\varepsilon_2 > 0$

$\rightsquigarrow$ This means that ε_1 must be < 0 for a guided wave (so, ① must be a metal!)

Back to our wave equation:

$$-\frac{\partial^2 H_y}{\partial z^2} = \left\{\frac{\omega^2 \varepsilon_r}{c_0^2} - k^2\right\} H_y$$

now $$\frac{\partial^2 H_y}{\partial z^2} = \frac{1}{\delta_i^{\,2}}$$

$$\Rightarrow \quad \frac{1}{\delta_2^{\,2}} = -\left(\frac{\omega^2 \epsilon_2}{c_0^2} - k^2\right)$$

$$\& \quad \frac{1}{\delta_1^{\,2}} = -\left(\frac{\omega^2 \epsilon_1}{c_0^2} - k^2\right)$$

$$\frac{\delta_1^{\,2}}{\delta_2^{\,2}} = \frac{\epsilon_2^{\,2}}{\epsilon_1^{\,2}} = \frac{\left(\frac{\omega^2 \epsilon_2}{c_0^2} - k^2\right)}{\left(\frac{\omega^2 \epsilon_1}{c_0^2} - k^2\right)}$$

~ rearrange:

$$\frac{1}{\epsilon_1^{\,2}}\left(\frac{\omega^2 \epsilon_1}{c_0^2} - k^2\right) = \frac{1}{\epsilon_2^{\,2}}\left(\frac{\omega^2 \epsilon_2}{c_0^2} - k^2\right)$$

$$\left(\frac{\omega^2}{c_0^2 \epsilon_1} - \frac{k^2}{\epsilon_1^{\,2}}\right) = \left(\frac{\omega^2}{c_0^2 \epsilon_2} - \frac{k^2}{\epsilon_2^{\,2}}\right)$$

$$\frac{\omega^2}{c_0^2}\left(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2}\right) = k^2\left(\frac{1}{\epsilon_1^{\,2}} - \frac{1}{\epsilon_2^{\,2}}\right)$$

seek an expression that links ω & k

$$\frac{\omega}{c_0}\sqrt{\frac{\epsilon_2-\epsilon_1}{\epsilon_1\epsilon_2}} = k_1\sqrt{\frac{\epsilon_2^2-\epsilon_1^2}{\epsilon_1^2\epsilon_2^2}}$$

$$\Rightarrow k_1 = \frac{\omega}{c_0}\sqrt{\frac{\epsilon_2-\epsilon_1}{\epsilon_2^2-\epsilon_1^2}} \times \sqrt{\epsilon_1\epsilon_2}$$

$$\boxed{k_1 = \frac{\omega}{c_0}\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}} \Leftarrow \text{desired result}$$

~ This is a "dispersion" relation that shows how k changes w/ ω

~ not a linear relationship b/c ϵ_1 is a function of ω

c/f light: $k = \frac{n\omega}{c_0}$

linear dispersion relation

Use Drude model for metal

$$\epsilon_1 = \epsilon_{1r} + i\epsilon_{1i}$$

$$\epsilon_{1r} = 1 - \omega_p^2/\omega^2 \quad \& \quad \epsilon_{1i} = \omega_p^2/\omega^3\tau$$

~ for optical frequencies $\epsilon_{1r} \gg \epsilon_{1i}$

$$\Rightarrow \quad k \simeq \frac{\omega}{c_0}\sqrt{\frac{\epsilon_2(1-\omega_p^2/\omega^2)}{1+\epsilon_2-\omega_p^2/\omega^2}}$$

Look at limits! large k ($k \to \infty$)

$$1 + \epsilon_2 - \omega_p^2/\omega^2 \to 0$$

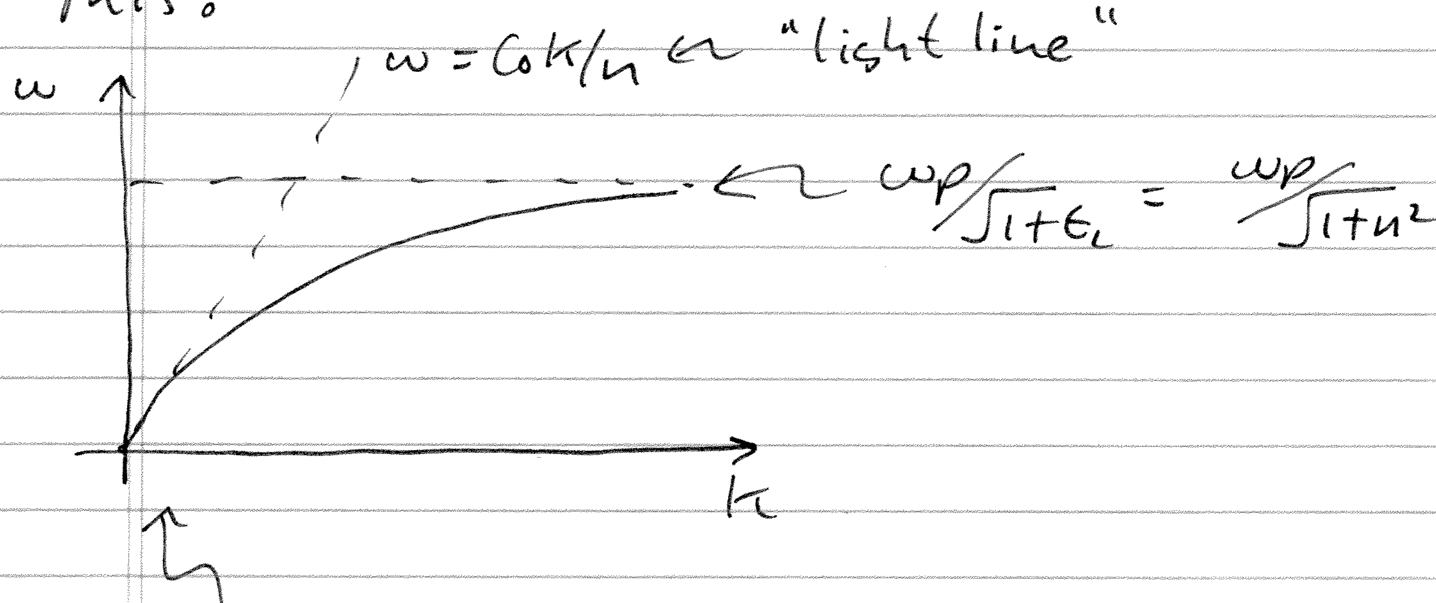
$$\Rightarrow \quad \omega = \omega_p/\sqrt{1+\epsilon_2}$$

~ small k ($k, \omega \to 0$)

$$k = \frac{\omega}{c_0}\sqrt{\frac{\epsilon_2(1-\omega_p^2/\omega^2)}{1+\epsilon_2-\omega_p^2/\omega^2}} \simeq \frac{\omega\sqrt{\epsilon_2}}{c_0}$$

$$\frac{\omega\sqrt{\epsilon_2}}{c_0} = \frac{\omega n}{c_0}$$ ~ "light line"

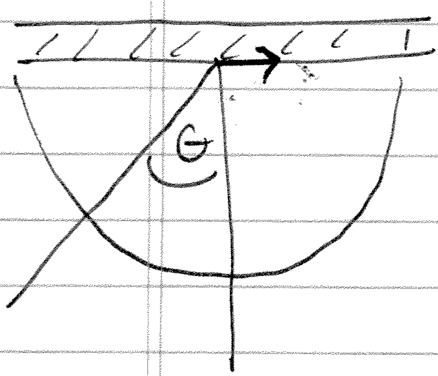
~ so the dispersion curve for "surface plasmons" (EM waves trapped at metal dielectric interfaces) looks like this:



wavevectors for SP & light dont meet ⇒ cant excite SPs directly by photons

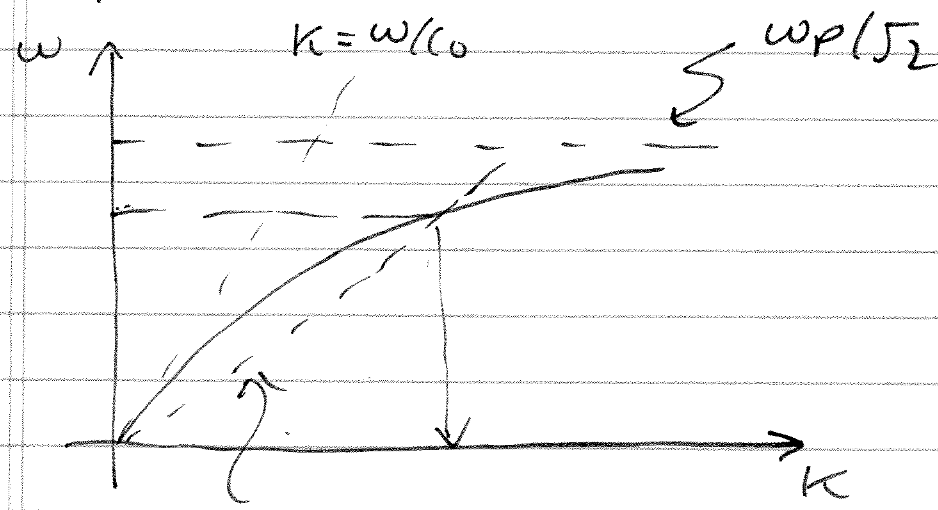
~ excite SPs in films by coupling through a prism

Example: medium 2 = air



$$k_x = k n_g \sin\theta$$

$$= \frac{\omega n_g}{c_0} \sin\theta$$



$k = \frac{\omega n_g}{c_0} \sin\theta$ ⟵ tuning the angle allows us to match k's at different ωs