Derivative Rules:

$c' = 0$

$(x^n)' = n x^{n-1}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\tan x)' = \sec^2 x$

$(\sec x)' = \sec x \tan x$

$(\csc x)' = -\cot x \csc x$

$(\cot x)' = -\csc^2 x$

$(c \cdot f(x))' = c f'(x)$

$(f(x) + g(x))' = f'(x) + g'(x)$

$(f(x) g(x))' = f'(x) g(x) + f(x) g'(x)$

$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) g(x) - f(x) g'(x)}{(g(x))^2}$

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$(e^x)' = e^x$

$(e^{f(x)})' = e^{f(x)} \cdot f'(x)$

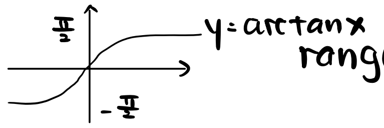
$(\ln x)' = \frac{1}{x}$

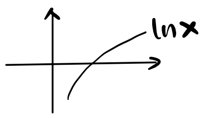
$(\ln f(x))' = \frac{1}{f(x)} \cdot f'(x)$

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \frac{1}{x^2+1}$

Practice :

1. a)  range : $[0, \frac{\pi}{2}]$ domain: $x \in (-\infty, \infty)$

b)  range: $(-\infty, \infty)$

domain: $0 < 3r + 5 < \infty \Rightarrow -\frac{5}{3} < r < \infty$

c) range: $(0, \frac{\pi}{2}]$

domain: $x \in (-\infty, 0]$ $\sin x \in [-1, 1]$ $\arcsin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Rightarrow e^x \in [0, 1] \Rightarrow x \in (-\infty, 0]$

2. a) $y' = e^{\sin 2x} \cos 2x \cdot 2 + \cos e^{2x} e^{2x} \cdot 2$

b) $y' = e^{x^2 - x} (2x - 1)$

c) $f'(t) = a e^{t} \sin bt + e^{at} b \cos bt$

d) $g'(t) = \frac{1}{\arctan t^4} \frac{4t^3}{t^8+1} = \frac{4t^3}{\arctan t^4 (t^8+1)}$

3. a). $\ln y = \ln(x^{2x+1})$

$\ln y = (2x+1) \ln x$

$\frac{1}{y} \cdot y' = 2 \ln x + \frac{2x+1}{x}$

$y' = y(2 \ln x + \frac{2x+1}{x}) = x^{2x+1}(2 \ln x + \frac{2x+1}{x})$

b) $\ln y = \ln(\sin x^{x^2 - x}) = (x^2 - x) \ln \sin x$

$\frac{1}{y} \cdot y' = (2x-1) \ln \sin x + \cot x (x^2 - x)$

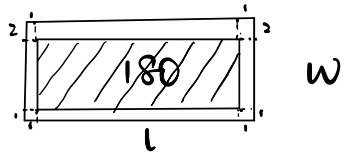
$\Rightarrow y' = \sin x^{x^2-x} \left((2x-1) \ln \sin x + \cot x (x^2 - x) \right)$

c) $\ln y = \ln((x-1)^{\ln x}) = \ln x \ln(x-1)$

$\frac{1}{y} y' = \frac{\ln(x-1)}{x} + \frac{\ln x}{x-1}$

$y' = (x-1)^{\ln x} \left(\frac{\ln(x-1)}{x} + \frac{\ln x}{x-1} \right)$

4.

a) 

b) $(w-3)(l-2) = (w-3)\left(\frac{180}{w}-2\right)$

c) $\frac{d}{dw}\left((w-3)\left(\frac{180}{w}-2\right)\right) = \frac{540}{w^2} - 2 = 0$

$\Rightarrow w = 3\sqrt{30} \approx 16.43$

d) $(3\sqrt{30}-3)\left(\frac{180}{3\sqrt{30}}-2\right) = 186 - 12\sqrt{30} \approx 120.27$