CITATION

TITLE:	Synthese, 128
AUTHOR:	Shrader-Frechette
PUBLISHER	Kluwer Academic Pub
YEAR	2001
PAGES	319-342
File #	020702003_Shrader

KRISTIN SHRADER-FRECHETE

USING A THOUGHT EXPERIMENT TO CLARIFY A RADIOBIOLOGICAL CONTROVERSY

ABSTRACT. Are philosophers of science limited to conducting autopsies on dead scientific theories, or might they also help resolve contemporary methodological disputes in science? This essay (1) gives an overview of thought experiments, especially in mathematics; (2) outlines three major positions on the current dose-response controversy for ionizing radiation; and (3) sketches an original mathematical thought experiment that might help resolve the low-dose radiation conflict. This thought experiment relies on the assumptions that radiation "hits" are Poisson distributed and that background conditions cause many more radiation-induced cancers than human activities. The essay closes by responding to several key objections to the position defended here.

1. THOUGHT EXPERIMENTS

Thought experiments are ways of exploring factual reality through reasoning. Nicholas Rescher (1991) argues that they are the characteristic method of the Greek nature philosophers, and everyone has heard of the wonderful thought experiment in Galileo's *Discorsi* (1967), in Einstein's chasing a light beam (Einstein 1949), and in using Schrödinger's cat to ridicule the orthodox formulation of the quantum formalism. Even in contemporary physics, thought experiments remain valuable. In his *Lectures on Physics*, Richard Feynman praises Stein's sixteenth-century thought experiments on the inclined plane. Remarking that "cleverness . . . is relative", Feynman notes that one can obtain the same (static-equilibrium) results "in a way which is even more brilliant, discovered by Stein and inscribed on his tombstone" (Feynman 1963, I, 4).

An essential characteristic of any thought experiment – including a mathematical thought experiment – is that it be an exploratory, ideal process to answer a theoretical question in the general framework of a given discipline and that it be carried out according to both the rules of logic and the particularities of the discipline itself. Thought experiments (especially mathematical thought experiments), however, need not have isomorphic counterparts in the area of laboratory-like or field-like experiments, and for at least two reasons. *First*, many thought experiments (for example,

Synthese **128:** 319–342, 2001.

in physics) involve non-imitable idealizations of actual conditions under which phenomena take place. *Second*, in the case of mathematical thought experiments, like variants of the one examined in this paper, there are no laboratory-like counterparts at all. Indeed, some say that the only "genuine experiments" in mathematics are thought experiments (Anapolitanos 1991, 87).

But if thought experiments need have no empirical counterparts, then how can they have novel empirical or mathematical import, since they take place entirely inside one's head? One answer is that they are arguments, not some special window on the world (see Brown 1991). As arguments, they (i) posit hypothetical or counterfactual states of affairs and (ii) invoke particulars irrelevant to the generality of the conclusion (Norton 1991, 129; 1996, 333–336).

The concern of this paper, however, is neither precisely what thought experiments are, nor how they can be justified, nor whether the logic used in them has a privileged status, as Frege supposed (see Massey 1991). Instead, this paper asks: (A) If one examines a particular mathematical thought experiment in radiation physics, what type of thought experiment is it? (B) To what degree does this thought experiment conform to standard constraints prescribed for mathematical thought experiments? (C) What import might this mathematical thought experiment have for helping to resolve the current controversy over the dose-response curve for ionizing radiation?

Thought experiments have several distinguishing characteristics and can be categorized in a number of ways (see Irvine 1991, 159). One crude classification is into *refuters*, *corroborators*, and *clarifiers*. Karl Popper (1959) calls the refuters "critical" thought experiments, and the corroborators, "heuristic" thought experiments. *Refuting* thought experiments provide counterexamples that try to overturn statements by disproving one of their consequences. Refuting thought experiments are typically *reductio ad absurdum* arguments (see Brown 1991, 76ff.; Norton 1991, 131). *Corroborating* thought experiments provide imaginative analogies that aim at substantiating statements, as in the famous abortion arguments by Judith Jarvis Thomson (1971). Unlike corroborating thought experiments, *clarifying* thought experiments provide imaginative analogies that aim neither to refute nor to corroborate, but to illuminate some case, as did Ezra Mishan (1972, 21). In order to clarify the choice whether to build an airport nearby, or farther away at an additional cost of $2 million per year, Mishan proposed a thought experiment: dividing the annual cost of the distant relocation by the number of residents x who would avoid noise pollution from the nearby location. If nearby residents asked whether it was worth \$2 million/x (or approximately \$20 per year per household) to avoid the closer location, Mishan said that this thought experiment would clarify the airport controversy and make it easier to resolve. Because the mathematical thought experiment (discussed in this paper) seems both to corroborate the claim that the dose-response curve for low-level ionizing radiation is linear with no threshold and to clarify the controversy over radiation, it seems to be both a corroborating and a clarifying thought experiment in mathematics.

2. MATHEMATICAL THOUGHT EXPERIMENTS, APPLICABLE TO RADIATION PHYSICS

Standard work on thought experiments in mathematics divides them into at least 6 groups. (1) Presupposing a new conceptual framework, some mathematical thought experiments attempt to answer specific questions as to whether something is the case or not, e.g., whether the well-known formula – relating the number of vertices, edges, and faces of a regular polyhedron – is provable. (2) Other mathematical thought experiments also attempt to answer specific questions as to whether something is the case or not, but they do so within the strict framework of fixed theory, as when one asks, for example, whether the Axiom of Choice or its negation is provable from the axioms of the Zermelo-Fraenkel set theory. (3) Still other mathematical thought experiments arise during a period of foundational crisis and attempt to construct a new conceptual framework, as when mathematicians in the beginning of the twentieth century proposed various modifications of the naive Cantorian concept of the set in order to address set-theoretic paradoxes. (4) A fourth type of mathematical thought experiment emerges when thinkers attempt to corroborate or refute some postulate that has come to be accepted as basic, but which seems impossible to prove or disprove, as when geometers in the beginning of the nineteenth century tried to negate the parallel postulate. (5) Another distinct type of mathematical thought experiment arises when curious mathematicians or philosophers, working within normal science, discover a way to reconceptualize something, quite by surprise, as when Robinson rehabilitated Leibnizian infinitesimals in the middle of this century. (6) A final type of mathematical thought experiment occurs when thinkers attempt to devise a new mathematical framework that, while not revolutionary, is easier to employ, as exemplified by the recent adoption of infinitary combinatorics (Anapolitanos 1991, 88–94).

According to the preceding six-part classification, the mathematical thought experiment in radiation physics (to be discussed here) likely falls into category (2). It is designed to answer a specific question (is the dose-response curve for ionizing radiation linear with no threshold at low exposures?), and it does so within the set of 6 fixed assumptions accepted by virtually all parties to the dose-response conflict. As later paragraphs will show, because the mathematical thought experiment neither presupposes nor intends to support a new conceptual framework, yet it aims to illuminate a specific question, it appears to fit easily within category (2) of mathematical thought experiments.

A mathematical, rather than an actual, thought experiment is essential to illuminating the radiation controversy because the shape of the dose-response curve for ionizing radiation is currently empirically underdetermined. There is no compelling epidemiological evidence about low-dose effects of ionizing radiation, and DNA techniques (that tie specific molecular responses to different radiation exposures) are not yet well enough developed to specify the curve. As a result, there are an infinite number of mathematical functions (each with different assumptions about behavior at low doses) that pass through all the data points representing observations of radiation effects at high doses. Also it is difficult to obtain person-specific, radiation-exposure estimates. One reason is differences among filters. The U.S. Environmental Protection Agency, for example, uses filters that detect only about 15 percent of the atmospheric radioiodine that the Finns detect in their filters (Caufield 1989, 238–239). Another reason is the presence of many global, undetected, hot spots, with radiation levels millions of times above average (Robbins, Makhijani, and Yi 1991, 16–17). In addition, sample sizes necessary for low-dose studies would have to be extraordinarily large – and the follow-up time extremely long – in order for epidemiological and statistical methods to detect low-probability effects of radiation, such as cancers. But as the sample sizes increased, the likelihood of population exposure to other toxicants would increase and confuse the results. High naturally occurring rates of cancer and individual variations in nutrition, lifestyle, and genetic susceptibility also obscure empirical effects of low-dose ionizing radiation (Trosko 1996, 812; Schull 1995, 277). Besides, there is no unique "fingerprint" in the DNA from radiation-induced, versus other, genetic disturbances (Trosko 1996, 815–817), and no compelling, consensus-based biological model of radiation carcinogenesis (Schull 1996, 800). For all these reasons, experiments alone currently are unlikely to settle the conflict over the radiation dose-response curve.

Other problems suggest that it may be reasonable to try to employ mathematical thought experiments – and not just experiments – to clarify the shape of the radiation dose-response curve. Most radiation studies are able to control neither for external and internal selection effects nor for variation in susceptibility with age at exposure. Studies that stratify exposed populations for age at exposure show much higher and statistically-significant risks from low-level ionizing radiation, while those that ignore age stratification do not, in part because of the "healthy-worker effect" (Stewart and Kneale 1993; Nussbaum and Kohnlein 1995, 202–204). Some researchers, for example, compare deaths of radiation workers to fatalities in the general population and then conclude that low-level radiation causes no additional health effects. But other physicists compare deaths of radiation workers with fatalities for comparable young and healthy groups. As a consequence, they conclude that low-level radiation causes significant health effects. Despite the conflicting results, radiobiologists do not agree on which, if either, way to study radiation effects is superior. Because they do not agree on proper controls, their experiments are unlikely to settle the dose-response controversy.

To devise a mathematical thought experiment that might clarify the low-dose controversy, it is important to define when radiation is low-dose. Although the same dose affects various tissues and people differently, some physicists believe that a low dose is what causes only one particle track across a nucleus. According to this definition, a low dose is less than 0.2 mGy (20 millirads) or less than one-tenth of the average annual background dose (200 millirads). Because virtually all physicists agree that a low dose is something under 200 mGy (20 rads) per year (Fry 1996, 823), the mathematical thought experiment developed and evaluated in this paper likewise presupposes low doses are exposures at (and below) 20 rads.

In addition to presupposing the preceding definition of "low dose", the mathematical thought experiment (developed here) also aims to satisfy a number of theoretical conditions. Any list of conditions for all mathematical thought experiments must be open-ended, because there is no complete agreement on a precise definition of such conceivability constraints. Nevertheless, several of the more important conceivability constraints, for mathematical thought experiments, include (1) simplicity conditions, (2) familiarity conditions, (3) plausibility conditions, (4) efficiency conditions, and (5) conceptualization conditions (see Anapolitanos 1991; Horowitz and Massey 1991). These constraints require, respectively, that the mathematical thought experiment (1) be clear, readily understood, and without any superfluous details; (2) be humanly tractable; (3) be believable enough to facilitate communication among mathematicians, philosophers, and scientists; (4) be able to be achieved, in a reasonable time, using both human and computer assistance; and (5) be able to be represented mathematically, just as Descartes, for example, was able to represent geometry algebraically.

Practically speaking, another important constraint on mathematical thought experiments is that, to the degree that they are intended to clarify some controversy or to corroborate some position in a given controversy, their conceptual framework and starting point must be acceptable to all parties involved in the controversy. To satisfy this practical constraint, the mathematical thought experiment (to be developed here) will begin with some noncontroversial assumptions, likely to be accepted by all parties to the dose-response controversy. To understand these assumptions, however, one first must understand what commitments separate disputants in the low-dose conflict.

3. CONTROVERSY OVER THE DOSE-RESPONSE CURVE FOR IONIZING RADIATION

To illustrate the deep scientific conflict over the dose-response curve for low-level radiation, consider how their diverse accounts of the curve affect projected Chernobyl fatalities. On the one hand, the Soviets, the French, UN agencies, and many proponents of nuclear power tend to claim that the consequences of the Chernobyl reactor explosion and fire were minimal. They say Chernobyl caused only 28 casualties, although it is possible that latent cancers later may appear (MacLachlan 1994, 11ff). The International Atomic Energy Agency (IAEA), a UN group dominated by the nuclear industry, places the number of Chernobyl fatalities at 31, with possible later cancers still to appear (1991, 4). On the other hand, many health experts, scientists, and environmentalists, especially in developed nations, have argued that the Chernobyl effects were catastrophic. The U.S. Department of Energy says the accident has caused 32,000 premature deaths so far, not including cancer fatalities in later generations (Shcherbak 1996, 46). The Ukrainian government puts its national fatalities alone at 125,000 (Campbell 1996). John Gofman, a well-known medical doctor and environmentalist from the University of California, Berkeley, puts future Chernobyl-caused, premature deaths induced by germline mutations/cancer at 500,000 and future Chernobyl-induced nonfatal cancers at 475,000 (Gofman 1995).

One reason for such massive disagreement about the consequences of Chernobyl is that the IAEA – with its conservative death toll of 31 – relied on Soviet estimates of exposure levels, visited only two mildly contaminated villages, then failed to consider the 800,000 liquidators (clean-up personnel, mainly young military men) who had the highest exposures. It closed the International Chernobyl Project (ICP) epidemiological studies only three years after the accident, then concluded that there were "no health disorders that could be attributed directly to radiation exposure" (IAEA 1991, 508–510).

A second reason for the controversy over Chernobyl fatalities is that most will be neither immediate nor obvious. They are "statistical casualties", premature deaths that scientists infer from a radiation dose-response curve. This curve is based on a mathematical model that ties different amounts of radiation exposures to various health effects. Using Hiroshima and Nagasaki statistics, health physicists throughout the world agree (within an order of magnitude) about the shape of the dose-response curve at higher exposures. For very low doses, however, there is massive disagreement. If one assumes effects are linear, with a threshold for radiation damage at lower exposures, then (all things being equal) radiation-related health effects are likely to be minimal. As a consequence, (a) Chernobyl-induced premature cancer deaths may number only in the tens or hundreds; (b) governments may be able to deregulate low-level radioactive waste; and (c) ionizing radiation cannot have caused all the problems that atomic veterans, downwinders (near the Nevada nuclear test site), and radiation workers attribute to it (US DOE 1987). If effects are linear, without a threshold, then (all things being equal), radiation-related health effects are likely to be substantial. As a consequence, (a′) Chernobyl-induced premature deaths may number 500,000; (b′) governments may not be able to deregulate low-level radioactive waste; and (c′) ionizing radiation likely has caused numerous premature fatalities, especially among radiation workers and nuclear-weapons manufacturers (Gofman 1990, ch. 24). In other words, much of the answer to the divergent estimates of premature Chernobyl fatalities rests with different dose-response models for ionizing radiation. To develop a mathematical thought experiment that might help clarify this controversy, the main alternative positions on the dose-response curve must be clear. Once these are clear, it will be possible to show that the mathematical thought experiment (developed here) begins with presuppositions that virtually everyone would accept.

4. THREE MAIN POSITIONS ON THE RADIATION DOSE-RESPONSE CONTROVERSY

Most radiation physicists tend to subscribe to one of three positions – that the author calls "NT", "T", and "U". Position NT ("no threshold"), supported by the ICRP, the IAEA, and the U.S. National Academy of Sciences (NAS), is that even the lowest doses of ionizing radiation likely are risky. According to NT, the relationship between doses of ionizing radiation and health responses is linear, and any exposure increases the probability of harm. NT proponents argue that analysis of Hiroshima and Nagasaki data, as well as evaluation of the mortality of children irradiated *in utero*, support their position (Calabrese and Baldwin 1999; Nussbaum and Kohnlein 1995, 198–203; Trosko 1996, 818). They also point out that tumors almost always arise from single cells (Beninson 1996, 123; Bond et al. 1996, 878; Jones 1984, 539). As a result, NT proponents say, a single mutational event (radiation track) has a finite probability of generating DNA damage that can cause a tumor. This probability is not zero, say NT proponents, because less than 0.2 mGy (0.02 rad) – or about one-tenth the average annual dose from background radiation – is enough to cause a single particle track across a nucleus (Fry 1996, 823). Only 35 eV can alter a biological molecule (Bond et al. 1996, 880). Given this non-zero probability, NT proponents claim that most standard-setting bodies are correct to maintain that any apparent adaptation to low-dose ionizing radiation is "essentially short term" (Kovan 1995, NRPB 1995; NRC 1990; Jones 1984, 537). Instead, they argue that apparent repair of radiation damage creates cells that become like broken plates, glued back together. Just as glued plates are more likely to break again, NT advocates say "repaired" cells survive in a weakened state and are more likely to die from other causes (Caufield 1989, 159; UNSCEAR 1994; Gonzàlez 1994, 40).

Hypothesis T ("threshold"), supported by the French scientific community and the nuclear industry – contradicts the position of UNSCEAR, ICRP, NAS, and other groups. Thesis T is that ionizing radiation and some chemical toxicants are not harmful at low doses because they claim the body can offset the effects of small exposures. Proponents of T say it explains why some humans can receive fairly large amounts of radiation before they show signs of cancer (Cronkite and Musolino 1996). Other advocates of T maintain that low doses of radiation actually are beneficial and can increase factors such as fertility or growth rate (Sagan 1989; Luckey 1980; see Cohen 1987).

Still other physicists reject both NT and T. Instead, their hypothesis U is that measurement problems make the existence of a threshold for radiation damage currently unknowable or uncertain. Roger Clarke of the International Commission on Radiological Protection, Bo Lindell of the Swedish Radiation Protection Institute, Kenneth Mossman of the American Health Physics Society, and Gunnar Walinder of the Swedish Nuclear Training Center all support U. They argue that, whether or not exposures below 100 mGy (10 rads) are risky, their effects are too small to observe, purely speculative, and therefore unknowable (Clarke 1999; Lindell 1996b, 159;

Mossman 1999; Mossman 1996; Mossman et al. 1996; Walinder 1995; Greenhalgh 1996).

At present, much of the disagreement among NT, T, and U proponents arises because NT advocates typically depend on research based on long-term exposure data, age-stratified studies, large sample sizes, or absence of caloric restriction in test subjects. T and U advocates, however, tend to rely on short-term exposure data, non-stratified studies, small sample sizes, or test subjects under caloric restrictions (see MacLachlan 1995, 1ff.; Duport 1996). In part because each group relies on different methods and data, they are able to arrive at different conclusions regarding the dose-response curve for ionizing radiation. In order to help clarify the dose-response controversy, however, this paper's mathematical thought experiment must avoid begging any questions of method, data, or framework. Instead, its starting points must rely on assumptions acceptable to all parties to the NT, T, or U controversy. Otherwise, the mathematical thought experiment will do little to clarify the controversy or to corroborate a particular position about it.

5. SHARED ASSUMPTIONS AMONG PROPONENTS OF NT, T, AND U

A starting point for any mathematical thought experiment (to clarify the radiation-curve controversy) is to ask what assumptions proponents of all three positions – NT, T, and U – might hold in common. For example, they disagree as to whether ionizing radiation (from human sources) produces cancer by the same mechanism as background radiation (Wilson 1996, 19; Crump et al. 1976). Nevertheless, there appear to be at least six crucial points on which proponents of NT, T, and U agree. These propositions might provide a starting point for a mathematical thought experiment on which all of the disputants could agree. One common assumption is (**A1**) that even the slightest amounts of ionizing radiation produce an ionization track through a cell, a track that theoretically is capable of producing cancer (see Myrden and Hiltz 1969; Modan et al. 1977, 1989; Boice and Monson 1977; Stewart and Kneale 1970; Harvey et al. 1985; see Gofman 1990, ch. 21). A second shared presupposition is (**A2**) that all repair of radiation-induced cell damage typically takes place within about 6 hours after exposure, or else it is not repaired (Brackenbush and Braby 1988, 256; Wilson 1996, 19). A third shared assumption is (**A3**) that cancer begins in a single cell, and mutations cause cancers (Beninson 1996, 122-123; Bond et al. 1996, 878; Trosko 1996, 812; Fry 1996, 824-825; Jones 1984, 539). Proponents of all three positions also agree (**A4**) that exposures to radiation are cumulative, and any additional (human-caused) exposures never begin at zero. Because of background radiation throughout the world, everyone is subjected to a minimum of at least several hundred millirads per year. As a result, any additional (human-caused) radiation exposures, even for a newborn baby, never begin from a zero dose (see Wilson 1996, 19). Proponents of NT, T, and U likewise tend to agree (**A5**) that mutations require at least one ionizing hit in which a charged particle transfers energy to an object like DNA (see Bond et al. 1996, 877; Beninson 1996, 124; Trosko 1996, 812). Finally they all tend to assume (**A6**) that, according to simple target theory (see Kellerer 1996, 835; Urquhart 1987, 24; Beninson 1996, 123), the number of radiation hits (single ionizing events) in a critical volume (like DNA) over a given period or for a given dose of ionizing radiation, is Poisson distributed with probability

$$\text{(P1)} \qquad P(n) = e^{-x}x^n/n!$$

where x = the mathematical expectation (or average number) of hits in some time or space interval; where e is the base of the natural log system, 2.71828; and where n = the number of radiation hits (see Beninson 1996; Lindell 1996a; Walinder 1995).

6. USING THESE ASSUMPTIONS TO DEVELOP A MATHEMATICAL THOUGHT EXPERIMENT

Because virtually all radiobiologists and radiation physicists subscribe to the six preceding assumptions, (A1)–(A6), they may provide a basis for a mathematical thought experiment to clarify the radiation-dose controversy. The heart of the proposed mathematical thought experiment is hypothesis (P1) – assumption (A6) – that the number of radiation hits (single ionizing events) in a critical volume (like DNA), over a given period or for a given dose of ionizing radiation, follows a Poisson Distribution:

$$\text{(P1)} \qquad P(n) = e^{-x}x^n/n!$$

If (P1) is correct, then it follows that

$$\text{(P1A)} \quad P(1\text{ hit}) = e^{-x}x^1/n! = e^{-x}x$$

But if (P1A) is correct, then it also follows that

$$\text{(P1B)} \quad P(0\text{ hits}) = e^{-x}x^0/0! = e^{-x}(1)/1 = e^{-x}$$

is correct. And if (P1B) is correct, then it likewise follows that

(P1C) $P(\text{at least 1 hit}) = 1 - P(0\text{ hits}) = 1 - e^{-x}$

But if the standard assumptions (A1) through (A6) are correct, and if the deductions above, from (P1) through (P1C) also are correct, then (P2) represents the probability of at least one hit in the DNA:

(P2) $1 - e^{-x}$

But in order for cancer to arise, some scientists claim that at least 2 different target areas in the DNA must be hit, by 2 different particles. T proponents say up to 7 different target areas must be hit (Lindell 1996a, 3; Beninson 1996, 124; Kellerer 1996, 834). Still other scientists claim to have confirmed that one hit, in three different target areas, triggers cancer (Whitehead et al. 1999; Travis 1999). Despite their disagreement, proponents of hypotheses NT, T, and U likely would agree that the probability of n hits in different target areas is

(P3) $(1 - e^{-x})^n$

If (P1) through (P3) are plausible, then a mathematical thought experiment based on the simple relation (P3) may provide some insights into the role of radiation in carcinogenesis.

If R is the expectation of radiation-induced hits, as a function of time, and if M is the expectation of hits induced by all other causes, as a function of time, then the probability, over time, that radiation and other mutagens will hit at least n target areas in DNA is

(P4) $(1 - e^{-(R+M)})^n$

Of course, (P4) presupposes that radiation R and other mutagens M do not interact to induce mutations and cancers, and this presupposition could be false. Nevertheless, if one makes this and several other assumptions (that expectation of hits is a function of time and that the number of hits in a given volume, over time), is Poisson distributed (as (A6) presupposes), then a number of important results follow. Given (P4), and provided that n = at least 2, then the probability of radiation-induced cancers is given by

(P5) $\mathrm{PR} = (1 - e^{-(R+M)})^n - (1 - e^{-M})^n$

If (P5) is correct, then it might be possible to specify the probability of radiation-induced cancers, despite other causes of DNA damage.

To "check" her thought experiment, the author represented (P5) on a graphing calculator. (P5) appears linear with dose (or number of hits), at least for low doses, and at least when M is much larger than R. Consider the case in which M is 10 and R is 1, that is, in which hits induced by all other causes are 10 times greater than the hits induced by radiation. Substituting $M = 10$ and $R = 1$ in (P5), when the number of DNA target areas hit is n, and letting n vary from 1 through 25, it is clear that (P5) is linear. Using *Mathematica 3.0*, we obtain:

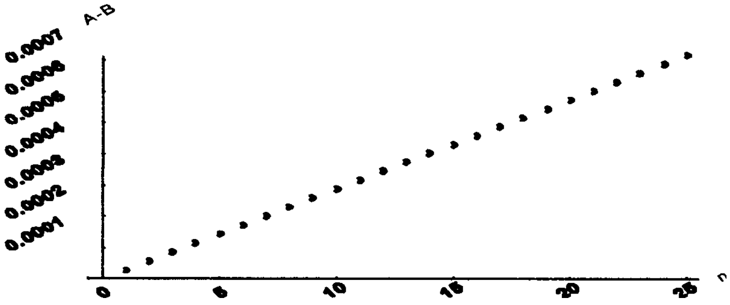

Figure 1.

Table 1 shows that, given the assumptions of Poisson Distribution, and that total other mutations M are much larger than radiation-induced mutations R, then the probability of radiation-induced cancers PR (A–B) is linear, with no threshold, at low doses. This particular variant (Table 1 and Figure 1) of the mathematical thought experiment, where $M = 10$ and $R = 1$ in (P5), is important because it is consistent with the fact that most experts believe radiation-induced mutations cause many fewer fatal cancers than all other mutations together. According to the United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR 1993a, 1993b, 1994; González 1994, 44; Lindell 1996a), radiation causes 1 in 40 of all fatal cancers. This case or variant (Table 1 and Figure 1) of the mathematical thought experiment also is significant because, if the simple mathematical thought experiment (P5) is close to correct, it provides an answer (yes) to the question whether the radiation dose-response curve is linear, with no threshold, at low doses. If the mathematical thought experiment is correct, then even 2 hits of radiation increase one's cancer risk. Besides, because of background exposures (see (A4)), everyone has at least 2 hits.

Following the preceding suggestion of UNSCEAR for the percent of cancers that is radiation-induced, consider the curve in which PR is linear with dose or number of hits. This is the case in which M is 40 and R is 1, that is, in which hits or mutations induced by all other causes are 40 times

TABLE I

n	A = $(1 - e^{-(R+M)})^n$	B = $(1 - e^{-M})^n$	A–B
1	0.9999833	0.9999546	0.0000286982
2	0.9999666	0.9999092	0.0000573947
3	0.9999499	0.9998638	0.0000860893
4	0.9999332	0.9998184	0.0000114782
5	0.9999165	0.999773	0.000143473
6	0.9998998	0.9997276	0.000172163
7	0.9998831	0.9996822	0.00020085
8	0.9998664	0.9996369	0.000229536
9	0.9998497	0.9995915	0.00025822
10	0.999833	0.9995461	0.000286902
11	0.9998163	0.9995007	0.000315583
12	0.9997996	0.9994553	0.000344261
13	0.9997829	0.99941	0.000372938
14	0.9997662	0.9993646	0.000401613
15	0.9997495	0.9993192	0.000430286
16	0.9997328	0.9992738	0.000458958
17	0.9997161	0.9992285	0.000487628
18	0.9996994	0.9991831	0.000516296
19	0.9996827	0.9991378	0.000544962
20	0.999666	0.9990924	0.000573626
21	0.9996493	0.999047	0.000602289
22	0.9996326	0.9990017	0.000630949
23	0.9996159	0.9989563	0.000659609
24	0.9995992	0.998911	0.000688266
25	0.9995825	0.9988656	0.000716921

greater than the hits or mutations induced by radiation. Using *Mathematica 3.0* and substituting $M = 40$ and $R = 1$ in (P5), Figure 2 and Table 2 show that, in this case, (P5) is linear with no threshold.

The thought experiment (just described) appears plausible, in part, because of some additional characteristics of the curve (P5) that make it a reasonable representation of the probability of radiation-induced cancers. When one looks at the slope of the curve (P5), for low levels of R, this

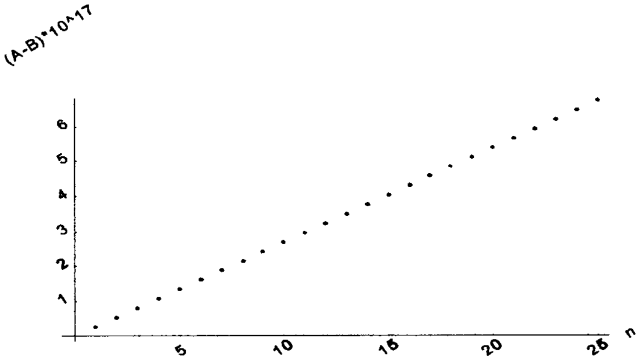

Figure 2.

TABLE II

n	A = $(1 - e^{-(R+M)})^n$	B = $(1 - e^{-M})^n$	A–B
1	0.9999999999999999984371178	0.9999999999999999957516457	$2.6854721 \times 10^{-18}$
2	0.9999999999999999968742356	0.9999999999999999915032915	5.370944×10^{-18}
3	0.9999999999999999953113534	0.9999999999999999872549372	8.056416×10^{-18}
4	0.999999999999999993748471	0.999999999999999983006583	$1.0741888 \times 10^{-17}$
5	0.999999999999999992185589	0.999999999999999978758229	1.342736×10^{-17}
6	0.999999999999999990622707	0.999999999999999974509874	$1.6112832 \times 10^{-17}$
7	0.999999999999999989059825	0.99999999999999997026152	$1.8798304 \times 10^{-17}$
8	0.999999999999999987496942	0.999999999999999966013166	$2.1483777 \times 10^{-17}$
9	0.99999999999999998593406	0.999999999999999961764812	$2.4169249 \times 10^{-17}$
10	0.999999999999999984371178	0.999999999999999957516457	$2.6854721 \times 10^{-17}$
11	0.999999999999999982808296	0.999999999999999953268103	$2.9540193 \times 10^{-17}$
12	0.999999999999999981245414	0.999999999999999949019749	$3.2225665 \times 10^{-17}$
13	0.999999999999999979682532	0.999999999999999944771395	$3.4911137 \times 10^{-17}$
14	0.999999999999999978119649	0.99999999999999994052304	$3.7596609 \times 10^{-17}$
15	0.999999999999999976556767	0.999999999999999936274686	$4.0282081 \times 10^{-17}$
16	0.999999999999999974993885	0.999999999999999932026332	4.296755×10^{-17}
17	0.999999999999999973431003	0.999999999999999927777978	4.565303×10^{-17}
18	0.999999999999999971868121	0.999999999999999923529623	4.83385×10^{-17}
19	0.999999999999999970305238	0.999999999999999919281269	5.102397×10^{-17}
20	0.999999999999999968742356	0.999999999999999915032915	5.370944×10^{-17}
21	0.999999999999999967179474	0.999999999999999910784561	5.639491×10^{-17}
22	0.999999999999999965616592	0.999999999999999906536206	5.908039×10^{-17}
23	0.99999999999999996405371	0.999999999999999902287852	6.176586×10^{-17}
24	0.999999999999999962490827	0.999999999999999898039498	6.445133×10^{-17}
25	0.999999999999999960927945	0.999999999999999893791144	6.71368×10^{-17}

slope becomes independent of R and depends on M. For realistic values (because it represents the probability of fatal cancer) of

$$\text{(P6)} \quad (1 - e^{-(R+M)})^n$$

between 0.1 and 0.6, there is only slight variation in the slope. Moreover, the slope of (P5) has a maximum when the risk PR (A–B) is 1.1. This maximum is consistent with the fact that the cancer risk from mutagens other than radiation – namely

$$\text{(P7)} \quad (1 - e^{-M})^n$$

may increase rapidly with exposure (e.g., with years of life) but, at some level, it must stop increasing because the total probability of cancer risk cannot exceed 1 (see Lindell 1996a, 3–4). But this last constraint means that the relationship expressing radiation-induced cancer risk – (P6) less (P7) – or PR, as a function of radiation-induced hits, R, is sigmoid. That is, if one holds n constant at 1 and substitutes, respectively, $R = 1, 2, \ldots, 25$ and so on, PR (A–B) remains sigmoid. Moreover, given that the total cancer risk (from radiation and other mutagens) is about 0.25 and slowly rising, it is reasonable to assume that we are on the middle part of the sigmoid curve (between 0.1 and 0.6), where the slope is fairly constant. But if we are on this middle part of the curve, then any exposure increment, such as from radiation, therefore causes two things: (1) a proportional risk (which is always the case within differential intervals) and (2) approximately one and the same risk per unit of exposure (dose). Thus, the mathematical thought experiment appears to have at least an initial plausibility.

Of course, to use this mathematical thought experiment, one must presuppose that the expectation of hits is a function of time, that the number of hits follows a Poisson Distribution, and so on (assumptions A1 through A6). The author remains troubled by the fact that one must also presuppose (in this thought experiment) that there is no significant interaction (such as synergy) among radiation and non-radiation means of inducing cancers and mutations, and it is not obvious whether this presupposition is borne out in reality.

7. GENERAL OBJECTIONS TO THE MATHEMATICAL THOUGHT EXPERIMENT

A potential problem with the mathematical thought experiment, sketched out in preceding pages, is both that it is fairly simple and also that it may include a doubtful presupposition (that there is no significant interaction (such as synergy) among radiation and other means of inducing cancer and mutations). Nevertheless, all participants in the dose-response controversy appear to agree on the 6 assumptions (A1–A6), already stated. Even so, the preceding thought experiment appears potentially vulnerable to three general types of objections: (1) Thought experiments may trivialize the problem they are meant to solve by begging the question at issue (see Sorensen 1992, 256–259). (2) As Bernard Williams notes, because contradictory thought experiments are possible, any particular thought experiment may frame a question in a way that predisposes the reader/hearer to agree with it, perhaps by overweighting familiar facts (Sorensen 1992, 261–269). And (3) because thought experiments are purely hypothetical, they may not provide conclusive evidence for a particular hypothesis or theory (see Wilkes 1988).

The most troubling objection to any thought experiment (and the fallacy most often compromising thought experiments, like that of Newton's buckets or Judith Jarvis Thomson's musician) appears to the be first, that the thought experiment begs the question. Does the mathematical thought experiment here beg the question in any important sense? To evaluate this objection one must determine whether either Equation (P2) or (P3) is linear with no threshold, the conclusion that the mathematical thought experiment hopes to support. Graphing (on a TI-86), the author was able to determine that, when x ranges from 1 to 7, Equation (P2) rises quickly. At about $x = 7$, it quickly becomes asymptotic and forms a horizontal line. Equation (P2) clearly is not linear. Similar, graphing quickly on a TI-86, the author was able to determine that in the case when $n =$ at least 2 (as most radiation physicists seem to agree), and when x ranges from 1 to 7, Equation (P3) rises quickly. However, in the case when $n =$ at least 2, and x ranges between 1 and 7, (P3) does not rise so rapidly as Equation (P2); at about $x = 7$, Equation (P3) quickly becomes asymptotic and forms a horizontal line. Thus Equations (P2) and (P3) are not linear. Because the thought experiment suggests that the curve representing risk from low-dose radiation is linear with no threshold, the author believes that there is no obvious sense in which the proposed mathematical thought experiment begs the question.

Another potential problem is whether the experiment is unrealistic in some damaging sense. Someone might object, for example, that because the equation (P5) – used as the mathematical thought experiment – has its maximum at 1.1 and not 1.0, and yet represents a probability, it is unrealistic. Another objection might be that (P5) presupposes there is no interaction among non-radiation hits and radiation-induced hits causing mutations and cancer. One response is that, because thought experiments are so simple, they systematically err under certain conditions, yet these errors are not necessarily a problem. For example, a compass is a simple but useful device for determining direction, even though it errs in the presence of magnets. Its scope is limited, and it becomes unreliable near the North Pole, in mineshafts, when vibrated, and when it is near metal. Also, the compass does not point precisely north, only close enough to north for most navigational problems. Moreover, most people who follow compasses likely do not know how or why they work (Sorensen 1992, 288–289). Nevertheless people use them. Employing analogous reasoning, the author believes that one might use this mathematical thought experiment, even with the knowledge that (like the compass), it is limited. Such limitations are less troublesome because, within the next several decades, DNA techniques are likely to enable molecular biologists to track the smallest amounts of radiation damage, independent of all the uncertainties surrounding epidemiological effects. As a result, provisional acceptance of (P5) does not appear particularly problematic.

In response to charge (2), bias or lack of realism in the thought experiment, it is important to note that the mathematical thought experiment does not frame the question of the radiation dose-response curve in a prejudicial or realistic way. These frames mainly include the assumptions (A1 through A6) already mentioned in earlier sections. Assumption A6, for example, about Poisson distribution, appears to be reasonable because it is part of most other cancer models. The presupposition that “hits” of ionizing radiation increase as a function of time also seems highly plausible because older people bear more evidence of exposure to ionizing radiation. Also, probability of cancer, all things being equal, increases with age. The author believes that the fundamental assumptions built into the thought experiment merely presuppose that the world is similar enough that reasoning about it sometimes works. As Simon Blackburn (1993, 10) puts it: “the world is not so disconnected that anticipation and imagination always fail, and we could not survive in it if it was”.

Regarding the classic objection (3) that thought experiments are purely hypothetical, the author believes that this mathematical thought experiment is not hypothetical in any damaging sense. After all, for this objection to succeed, the objector ought to show not merely that the mathematical thought experiment is hypothetical, but that it is hypothetical in some damaging sense. That is, objections of type (3) ought not reflect merely an aesthetic preference for true “stories” (see Sorensen 1992, 275–276). One reason is that reasonable people refuse to deliberate about improbable contingencies only when the stakes are low. The stakes are not low in the radiation case, as the Chernobyl discussion (earlier) indicates. Because the potential health risks are great, the charge that the thought experiment is hypothetical may be less important than whether it is hypothetical in some damaging sense. Another reason is that philosophers have long accepted "hypothetical" thought experiments. In his later work, Wittgenstein was addicted to examples and parables that one might call philosophical "thought experiments". His later method is largely one of exploring the phenomena by imagining changes and distortions, and then evaluating what happens as a result of these changes (see Blackburn 1993, 11; Gale 1991, 301). The value of such imaginings is that they may allow a new way of thinking about the radiation conflict. Like a sensitivity analysis, this thought experiment may not resolve the controversy, but it may contribute to its resolution, perhaps through clarification of the problem. After all, if actual experimental data always were conclusive, then there would be no need for thought experiments. This mathematical thought experiment is important because it appears to help clarify the problem of the radiation dose-response curve.

8. SPECIFIC OBJECTIONS TO VARIANTS OF THIS MATHEMATICAL THOUGHT EXPERIMENT

One specific objection to the mathematical thought experiment provided here is that, although there are grounds for assuming that the probability of mutations is proportional to the number of radiation hits (P1 through P5), there appears to be little reason to assume that the probability of cancers is proportional to the number of radiation hits. Although this objection appears theoretically plausible, it is not compelling on empirical grounds. There is empirical confirmation that only 3 hits in different DNA target areas are sufficient to produce cancer (Whitehead et al. 1999; Travis 1999), and because a hit of only 35 eV is sufficient to damage the DNA, it is clear that everyone has experienced DNA damage from background radiation. Thus, the number of hits is very large (and potential DNA damage could be quite large). Because of the large number of hits, and because cancer increases with age, just as the number of hits increases, it is reasonable to assume that the number of cancers is proportional to the number of hits.

Another person might claim that the mathematical thought experiment just outlined is "just a model", and not a real thought experiment. There are at least three responses to this model objection, MO. A *first* response is that, if the objector claims that any mathematical thought experiment is "just a model", then his objection may call into question all mathematical thought experiments, despite significant philosophical work on them (see, for example, Anapolitanos 1991; Brown 1991; Horowitz and Massey 1991; Massey 1991). Such an objection fails because it proves too much.

A *second* response to MO is that thinking about a mathematical or a physical model and *how it would behave* (as when one considers the moves from Equations (P1) through (P7)) is not the same as *manipulating* a mathematical or physical model and seeing *how it (in fact) behaves*. Reflection and execution are different. Just because one thinks about how a mathematical model would behave, and checks some part of one interpretation of it on a graphing calculator, does not mean that one is not doing a thought experiment. The partial "checks" merely contribute to the plausibility of the mathematical thought experiment – the conceptual relationships among (P1) through (P6). The heart of the thought experiment is not a check of one or more substitutions in one of the equations. Moreover, some thought experiments involve models, and others do not. If a thought experimenter thinks about the relationship between A and B in order to understand the relationship between C and D, then A and B may constitute a model for C and D. However if a thought experimenter thinks about the relationship between A and B, or postulates something about A and B in order to learn more about them, there may be no model involved. Thus, even if there is a model involved, in this mathematical thought experiment, this does not mean there is no thought experiment. Instead, it means merely that it is a thought experiment that employs a model of something in order to learn about it. Not all models involve thought experiments. For example, a model may not involve a thought experiment if it relies merely on simulation and not on thinking about relationships within the model and their consequences. And not all thought experiments involve models. For example, a thought experiment may not involve a model if no vehicles (such as A and B) are used as ways to understand something (such as C and D).

A *third* response to the model objection (MO) is that one would do well to distinguish among thought experiments, models, simulations, and re-enactments. If Sorensen (1992, 225–228) is correct, then MO confuses indirect thought experiments with models. Just because something is indirect does not mean that it is merely a model and not a thought experiment. Sorensen's example of a direct thought experiment is thinking about molecules of a solid when heat is applied. His example of an indirect thought experiment is thinking about people trying to hold hands when they are violently jumping up and down. The more violent the jumping, the harder it is to stay connected (Sorensen 1992, 225). Just because Sorensen uses the jumping model, to understand molecules subjected to heat, does not mean that he has not used a thought experiment. He merely has used an indirect thought experiment that relies on the analogy between heated molecules and jumping people.

A *third* objection someone might make to the mathematical thought experiment proposed here is that it claims to help corroborate and clarify NT yet, according to the objector, if the body does not repair all radiation damage, then NT is the case, whereas if the body does repair all radiation damage, then T is the case. According to the objector, there is no need for the mathematical thought experiment, because whether T or NT is correct depends merely on whether repair (of radiation damage) is the case. There are at least two responses to this no-need objection (NNO). First, the author – along with virtually all members of the radiation-physics community – assumes thesis (A2) that all radiation repair takes place within 6 hours of damage, or else it is not repaired. Obviously there is some repair, and everyone agrees on this point. Obviously not all repair is complete, and everyone agrees on this point. The issue is how extensive the repair is, and how much of it gets done in the initial 6 hours. Thus the author does not beg the question of repair, as NNO suggests, but merely accepts the standard assumption (A2). Moreover, as already discussed, the repair situation is more complex than the two options (repair/no repair) presupposed in the NNO. A *second* response to this objection is that, even if the NNO were correct (that NT would be true if there were no repair, and T would be true if there were always complete repair), this fact would not be relevant to the mathematical thought experiment. The thought experiment is needed precisely because, apart from what is the case, empirical data – about the completeness of repair and its long-term effects – are at present unknown.

Some partial empirical work also suggests that all radiation repair is not complete, and that the thought experiment here may be correct. Crump et al. showed that, if carcinogenesis by an external agent acts additively with any already-ongoing process, then under almost any model, the response will be linear at low doses, provided that the extra risk is less than the spontaneous or background risk, and provided that the individual cancers arise from a single cell (Crump et al. 1976). This work provides apparent empirical support for parts of the mathematical thought experiment discussed here. What is interesting is that, if the Crump research is correct, then it shows (just as assumption (A3) presupposes) that the statistical nature of the dose-response curve is governed by the extreme tail of the response distribution. This tail makes any process of discrete events approximately linear at low doses. Even simpler than Crump's considerations and the earlier use of the relationship (P5) for PR is a quick examination of the Taylor series generated by f at a. If x is the total of non-radiation-induced plus radiation-induced cancers, and if a is the number of non-radiation-induced cancers, then $(x–a)$ is the number of radiation-induced cancers. When x is larger than a, and when $(x–a)$ is a very small quantity, then it is easy to see that all the non-linear terms of the Taylor series are close to zero, and that the function is approximately linear. For the plausibility of this simple (therefore perhaps uninteresting) Taylor-Series consideration, in favor of the linear and non-threshold nature of the dose-response curve, one appears to need to assume merely that the number of radiation-induced cancers $(x–a)$ is very small in proportion to those that arise from other causes. Although the point about the Taylor Series is trivial, it provides some support for the Crump data, that, in turn, lend plausibility to part of this mathematical thought experiment.

ACKNOWLEDGEMENTS

Thanks to participants in the philosophy of science colloquium at the University of Notre Dame and Northwestern University, to participants in the Swedish Radiation Protection Institute Colloquium, and especially to Jim Cushing, Neil Delaney, Don Howard, David Hull, Bo Lindell, Ernan McMullin, Lars Persson, and Jack Wilson, all of whom have given helpful comments on several earlier versions of this paper.

REFERENCES

Anapolitanos, D. A.: 1991, ‘Thought Experiments and Conceivability Conditions’, in T. Horowitz and G. Massey (eds), *Thought Experiments in Science and Philosophy*, Rowman and Littlefield, Savage, MD.

BELLE Newsletters. Biological Effects of Low Level Exposures (1992–1996). 5 Vols. School of Public Health, University of Massachusetts, Amherst, MA.

Beninson, D.: 1996, ‘Risk of Radiation at Low Doses’, *Health Physics* **71**(2), 122–125.

Blackburn, S.: 1993, ‘What If? The Uses and Abuses of Thought Experiments’, *Times Literary Supplement* **4707**(18 June): 10–11.

Boice, J. D. and Monson, R. R.: 1977, ‘Breast Cancer in Women After Repeated Fluoroscopic Examinations of the Chest’, *Journal of the National Cancer Institute* **59**(3), 823–832.

Bond, V. P., L. Wielopolski, and G. Shani: 1996, ‘Current Misinterpretations of the Linear No-Threshold Hypothesis’, *Health Physics* **70**(6), 877–882.

Brackenbush, L. W. and L. A. Braby: 1988, ‘Microdosimetric Basis for Exposure Limits’, *Health Physics* **55**(2), 251–255.

Brown, J. R.: 1991, *The Laboratory of the Mind*, Routledge, New York.

Calabrese, E. J.: 1994, *Biological Effects of Low Level Exposures – Dose-Response Relationships*, Lewis Publishers, Boca Raton, FL.

Calabrese, E. J. and L. Baldwin: 1999, ‘The Marginalization of Hormesis’, *Toxicologic Pathology* **27**, 187–194.

Campbell, P.: 1996, ‘Chernobyl’s Legacy to Science’, *Nature* **380**(6576): 653.

Caufield, C.: 1989, *Multiple Exposures: Chronicles of the Radiation Age*, Harper and Row, New York.

Clarke, Roger: 1999, ‘Control of Low-Level Radiation Exposure’, *Journal of Radiological Protection* **19**(2), 107–115.

Cohen, J. J.: 1987, ‘Conference on Radiation Hormesis: An Overview’, *Health Physics* **53**(5), 519.

Cronkite, E. P. and S. V. Musolino: 1996, ‘The Linear No-Threshold Model’, *Health Physics* **70**(6), 775–776.

Crump, K. S., D. G. Hael, C. H. Langley, and R. Peto: 1976, ‘Fundamental Carcinogenic Processes’, *Cancer Research* **36**(19), 2973–2979.

Crump, K. S. and H. J. Clewell: 1996, ‘Cancer and Non-Cancer Risk Assessment Should Be Harmonized’, *BELLE Newsletter* **5**(2/3), 2–4.

Duport, P.: 1996, ‘The Recommendations of the French Academy of Science on ICRP 60’, *Bulletin of the Canadian Radiation Protection Association* **17**(1), 18–21.

Einstein, A.: 1949, ‘Autobiographical Notes’, in P. Schilpp (ed.), *Albert Einstein: Philosopher-Scientist*, Open Court, La Salle, IL.

Feynman, R., R. Leighton, and M. Sands: 1963, *The Feynman Lectures in Physics*, 3 Vols. Addison-Wesley, Reading, MA.

Fry, R. J. M.: 1996, ‘Effects of Low Doses of Radiation’, *Health Physics* **70**(6), 823–828.

Gale, G.: 1991, ‘Some Pernicious Thought Experiments’, in Horowitz and Massey 1991, pp. 297–304.

Galileo: 1967, *Dialogue Concerning the Two Chief World Systems*, 2nd rev. ed. Translated by S. Drake. University of California Press, Berkeley.

Gofman, J. W.: 1990, *Radiation-Induced Cancer from Low-Dose Exposure: An Independent Analysis*, Berkeley, CA: Committee for Nuclear Responsibility, Inc.

Gofman, J. W.: 1995, ‘Foreword’, in Yaroshinskaya 1995, pp. 1–13.

González, A. J.: 1994, ‘Biological Effects of Low Doses of Ionizing Radiation: A Fuller Picture’, *IAEA Bulletin* **4**, 37–45.

Greenhalgh, G.: 1996, ‘Radiation and Cancer – New Moves in the Threshold Debate’, *Nuclear Engineering International* **40**(497), 14–15.

Harvey, E. B., J. D. Boice, Jr., M. Honeyman, and J. T. Flannery: 1985, ’Prenatal X-Ray Exposure and Childhood Cancer in Twins’, *New England Journal of Medicine* **312**(9), 541–545.

Horowitz, T. and G. Massey: 1991, *Thought Experiments in Science and Philosophy*, Rowman and Littlefield, Savage, MD.

International Agency for Research on Cancer (IARC) Study Group on Cancer Risk among Nuclear Industry Workers: 1994, ‘Direct Estimates of Cancer Mortality due to Low Doses of Ionising Radiation: An International Study’, *Lancet* **344**, 1039–1043.

International Atomic Energy Agency (IAEA): 1991, *International Chernobyl Project: Technical Report*, IAEA, Vienna.

Irvine, A. D.: 1991, ‘On the Nature of Thought Experiments in Scientific Reasoning’, in Horowitz and Massey 1991, pp. 149–166.

Jones, T. D.: 1984, ‘A Unifying Concept for Carcinogenic Risk Assessments: Comparison with Radiation-Induced Leukemia in Mice and Men’, *Health Physics* **47**(4), 533–558.

Kellerer, A. M.: 1996, ‘Radiobiology Challenges Posed by Microdosimetry’, *Health Physics* **70**(6), 832–836.

Kovan, D.: 1995, ‘NRPB Cuts Up the Cut-Off Theory’, *Nuclear Engineering International* **40**(497), 15.

Lindell, B.: 1996a, ‘The Case of Linearity’, *SSI News* **4**(1), 2–4.

Lindell, B.: 1996b, 'The Risk Philosophy of Radiation Protection', *Radiation Protection Dosimetry* **68**(3–4), 157–163.

Luckey, T. D.: 1980, *Hormesis with Ionizing Radiation*, CRC Press, Boca Raton, FL.

MacLachlan, A.: 1994, 'Official Russian Register Keeps Chernobyl Death Toll at 28', *Nucleonics Week* **35**(46), 11ff.

MacLachlan, A.: 1995, 'French Academy Rejects 1990 ICRP Norms as Not Based in Science', *Nucleonics Week* **36**(47), 1ff.

Massey, B. D.: 1991, 'Do All Rational Folk Reason as We Do? Frege's Thought Experiment Reconsidered', in Horowitz and Massey 1991, pp. 99–112.

Mathematica Version 3.0, Wolfram Research, Champaign, IL.

Mishan, E.: 1972, *Economics for Social Decisions*, Praeger, New York.

Modan, B., E. Ron, and A. Werner: 1977, 'Thyroid Cancer following Scalp Irradiation', *Radiology* **123**, 741–744.

Modan, B., A. Chetrit, E. Alfandary, and L. Katz: 1989, 'Increased Risk of Breast Cancer after Low-Dose Irradiation', *Lancet* **1**, 629–631.

Mossman, K. L.: 1996, 'Guest Editorial – The Linear, No-Threshold Model in Radiation Protection: The HPS Response', *HPS Newsletter* **24**(3), 2.

Mossman, K.: 1999, 'Trivialization of Radiation Hormesis', *SSI News* **7**(2), 13–15.

Mossman, K. L., M. Goldman, F. Mass, W. A. Mills, K. J. Schiager, and R. J. Vetter: 1996, 'Radiation Risk in Perspective', *HPS Newsletter* **24**(3), 3.

Myrden, J. A. and J. E. Hiltz: 1969, 'Breast Cancer Following Multiple Fluoroscopies during Artificial Pneumothorax Treatment of Pulmonary Tuberculosis', *Canadian Medical Association Journal* **100**(22), 1032–1034.

National Radiological Protection Board (NRPB): 1995, 'Risk of Radiation-Induced Cancer at Low Doses and Low Dose Rates for Radiation Protection Purposes', *Documents of the NRPB* **6**(1).

National Research Council (NRC), Committee on the Biological Effects of Ionizing Radiation and Board on Radiation Effects Research Commission on Life Sciences: 1990, *Health Effects of Exposure to Low Levels of Ionizing Radiation, BEIR V*, National Academy Press, Washington, DC.

Norton, J. D.: 1991, 'Thought Experiments in Einstein's Work', in Horowitz and Massey 1991, pp. 129–148.

Norton, J. D.: 1996, 'Are Thought Experiments Just What You Thought?', *Canadian Journal of Philosophy* **26**(3), 333–366.

Nussbaum, R. and W. Köhnlein: 1995, 'Health Consequences of Exposures to Ionising Radiation from External and Internal Sources: Challenges to Radiation Protection Standards and Biomedical Research', *Medicine and Global Survival* **2**(4), 198–213.

Popper, K.: 1959, *The Logic of Scientific Discovery*, Hutchinson, London.

Rescher, N.: 1991, 'Thought Experimentation in Presocratic Philosophy', in Horowitz and Massey 1991, pp. 31–42.

Robbins, A., A. Makhijani, and K. Yih: 1991, *Radioactive Heaven and Earth: The Health and Environmental Effects of Nuclear Weapons Testing In, On, and Above the Earth*, The Apex Press, New York.

Sagan, L.: 1989, 'Policy Forum Section – On Radiation, Paradigms, and Hormesis', *Science* **245**(4918): 574, 621.

Schrödinger, E.: 1957, *Science Theory and Man (formerly published under the title, Science and the Human Temperament)*, Dover Publications, New York.

Schull, W. J.: 1995, *Effects of Atomic Radiation: A Half-Century of Studies from Hiroshima and Nagasaki*, Wiley-Liss, New York.

Schull, W. J.: 1996, ‘Radioepidemiology of the A-bomb Survivors’, *Health Physics* **70**(6), 798–803.

Shcherbak, Y. M.: 1996, ‘Ten Years of the Chornobyl Era’, *Scientific American* **274**(4), 44–49.

Sorensen, R. A.: 1992, *Thought Experiments*, Oxford University Press, New York.

Stewart, A. M. and G. W. Kneale: 1970, ‘Radiation Dose Effects in Relation to Obstetric X-Rays and Childhood Cancers’, *Lancet* **1**, 1185–1188.

Stewart, A. M. and G. W. Kneale: 1993, ‘The Hanford Data: Issues of Age at Exposure and Dose Recording’, *PSR Quarterly* **3**, 101–111.

Thomson, J. J.: 1971, ‘A Defense of Abortion’, *Philosophy and Public Affairs* **1**, 47–68.

Travis, John: 1999, ‘Add Three Genes, Get One Cancer Cell’, *Science News* **156**, 139.

Trosko, J. E.: 1996, ‘Role of Low-Level Ionizing Radiation in Multi-Step Carcinogenic Process’, *Health Physics* **70**(6), 812–822.

U.S. Department of Energy, Committee on the Assessment of Health Consequences in Exposed Populations, Chairman: M. Goldman, L. Anspaugh, R. J. Catlin, J. I. Fabrikant and P. Gudiksen: 1987, Health and Environmental Consequences of the Chernobyl Nuclear Power Plant Accident, DOE/ER-0332. US DOE, Washington, DC.

United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR): 1993a, Report of the United Nations Scientific Committee on the Effects of Atomic Radiation, UN, New York.

United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR): 1993b, *Adaptive and Stimulatory Responses to Radiation in Cells and Organisms – United Nations General Assembly, 42nd Session of UNSCEAR*, A/AS.82/R.538. UN, Vienna.

United Nations Scientific Committee on the Effects of Atomic Radiation (UNSCEAR): 1994, *Sources and Effects of Ionizing Radiation – UNSCEAR 1994 Report to the General Assembly, with Scientific Annexes*, UN, New York.

Urquhart, J.: 1987, ‘Leukaemia and Nuclear Power in Britain’, in Jones and Southwood 1987, pp. 233–242.

Walinder, G.: 1995, *Has Radiation Protection Become a Health Hazard?* Nykoping, Swedish Nuclear Training and Safety Center, Sweden.

Whitehead, Robert A. et al.: 1999, *Nature*.

Wilkes, K. V.: 1988, *Real People: Personal Identity Without Thought Experiments*, Oxford University Press, New York.

Wilson, R.: 1996, ‘The Need for New Risk Default Assumptions’, *BELLE Newsletter* **5**(2/3), 18–20.

Yaroshinskaya, A.: 1995, *Chernobyl – The Forbidden Truth*, University of Nebraska Press, Lincoln.

Yushkov, P. I., T. A. Chueva, and N. V. Kulikov: 1993, ‘Effect of Increased Background of Ionizing Radiation on European Birch in the Zone of the Chernobyl Accident’, *Russian Journal of Ecology* **24**(5), 309–314.

O’Neill Family Professor of Philosophy and Concurrent Professor of Biological Sciences
336 O’Shaughnessy Hall
University of Notre Dame
Notre Dame
Indiana 46556
U.S.A.