Models and Computability
The Mathematics of Julia Knight

September 30 – October 2, 2022
University of Notre Dame

TALK ABSTRACTS

Uri Andrews (University of Wisconsin - Madison)

"Stacking Turtles"

I will describe a particular type of construction, which I'm calling stacking turtles, to build a computable structure. I'll show several examples of this type of construction within Julia Knight's work, including in a project joint with me where we examined what it takes to build the models of a strongly minimal theory. In particular, we showed that if T is strongly minimal and $T\cap \exists_{n+2}$ is uniformly Δ^0_n, then every countable model of T has.a computable copy.

Natasha Dobrinen (University of Notre Dame)

"Ramsey theory on homogeneous structures"

Ramsey theory on relational structures has been investigated ever since Ramsey proved his seminal theorem for colorings of k-sized sets of natural numbers. While a multitude of classes of finite structures have been shown to possess the Ramsey property, analogues for infinite structures have proven more elusive. Initiated by Sierpinski in the 1930's, it was not until D. Devlin's work in 1979 that the Ramsey theory of the rationals was completely understood; the Ramsey theory of the Rado graph was only completed in 2006 by Laflamme, Sauer, and Vuksanovic. Methods for Ramsey theory on finite structures are generally not sufficient for proving Ramsey properties of their infinite homogeneous counterparts. This talk will provide an overview of the current state of Ramsey theory of homogeneous structures, including work of various author combinations from among Balko, Barbosa, Chodounsky, Coulson, El-Zahar, Erdos, Hajnal, Hubicka, Komjath, Konecny, Laflamme, Masulovic, Nesetril, Nguyen Van The, Patel, Posa, Rodl, Sauer, Vena, Zucker, and the speaker.

Valentina Harizanov (The George Washington University)

"Complexity, structure, and language"

Classically isomorphic structures can have very different computability theoretic properties. When a complexity bound for some property of a structure is preserved under isomorphisms, we call the bound intrinsic. We are interested in intrinsic and nonintrinsic bounds. Often, we can describe complexity of an aspect of a structure syntactically – typically using computable infinitary formulas, or measure it semantically – typically using Turing degrees. This connection between definability and computability has been one of the main themes in computable structure theory. Julia Knight was at the forefront of the development of the ideas and techniques of this theory. Together with her students and a large network of national and international collaborators, she has produced a vast body of important results. We will present some of Julia's major results in computable structure theory, including some of our joint results.

Matthew Harrison-Trainer (University of Michigan)

"Describing algebraic structures"

Scott showed that for each countable structure there is a sentence of infinitary logic which characterises it up to isomorphism among all countable structures. Moreover, we can ask what the simplest Scott sentence for a structure is; this is the complexity of describing the structure. Julia Knight was a driving force in a program of looking at various algebraic structures and determining how complicated they are to describe, with the goal of improving our understanding of those structures. We will discuss the results of that program as well as open questions.

Meng-Che (Turbo) Ho (California State University)

"Free structures and limiting density"

Gromov asked what a typical group looks like, and he suggested a way to make the question precise in terms of limiting density. The typical finitely presented group is known to share some important properties with the non-abelian free groups. Julia Knight conjectured that the typical group satisfies a zero-one law and has the same first-order theory as the free group.

We generalize Gromov's notion and Knight's question to structures in an arbitrary algebraic variety (in the sense of universal algebra). We give examples illustrating different behaviors of the limiting density. Based on the examples, we identify sufficient conditions for the elementary first-order theory of the free structure to match that of the typical structure; i.e., a sentence is true in the free structure if it has limiting density 1.

This is joint work with Johanna Franklin and Julia Knight.

Karen Lange (Wellesley College)

"Integer Part Inspirations: Complexity of problems related to finding integer parts in real closed fields"

When I started my post-doc at Notre Dame, Julia introduced me to the notion of an integer part of a real closed field, a discrete subring in which every element of the field is distance less than one away from an element of the subring. We began work to determine how hard it is to compute an integer part of a given computable real closed field. While this question remains open, our work on integer parts led us down many interesting paths, from exploring root-taking in generalized power series fields to determining lengths of sums of well ordered sets in ordered groups. Here we'll take a tour through integer parts, their properties, and the many problems that they inspired.

Antonio Montalban (University of California - Berkeley) - COLLOQUIUM

"A Robuster Scott Rank"

The Scott rank was introduced in the 60's as a measure of complexity for algebraic structures. There are various other ways to measure the complexity of structures that give ordinals that are close to each other, but are not necessarily equal. We will introduce a new definition of Scott rank where all these different ways of measuring complexity always match, obtaining what the author believes is the correct definition of Scott Rank. We won't assume any background in logic, and the talk will consist mostly of an introduction to these topics.

Antonio Montalban (University of California - Berkeley)

"The game metatheorem."

We present a new metatheorem based on Ash and Knight's original eta-system priority argument. The previous modification of Ash and Knight's metatheorem by the author was more hands-on and slightly more general than the original. This new version is more abstract, less hands-on, less general, but much easier to apply. We will also mention a new topological version that is currently work in progress with Andrew Marks.