Locally solvable vector fields and Hardy Spaces

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Abstract: Work with Jorge Hounie

We define a new class of Hardy spaces $h^p_F(\mathbb{R})$, $0 < p \leq 1$, associated to a finite set $F \subset \mathbb{R}$ by considering atomic decompositions of a special kind. For most values of $p \leq 1$ this "new" Hardy spaces coincide with the localizable Hardy space $h^p(\mathbb{R})$ of Goldberg but for a discrete set of values $p = 1, 1/2, \ldots,$ we have $h^p_F(\mathbb{R}) \subsetneq h^p(\mathbb{R})$. Functional properties of $h^p_F(\mathbb{R})$ will be discussed, such as the multiplication invariance of functions in the Schwartz class and the characterization of dual spaces. As an application we will characterize the homogeneous solutions of a real analytic planar vector field satisfying the one-sided Nirenberg-Treves condition $(\mathcal{P}^+)$ with boundary value in $h^p_{F(L)}(\mathbb{R})$ in terms of L^p uniform boundeness of their traces, $0 < p \leq 1$.