Existence of smooth solutions of some Stefan type problems in the case of nonregular initial data

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The classical Stefan problem is a nonlinear free boundary value problem for a parabolic equation which describes a crystallized process. The substances can have a various phase station. The function of temperature has a constant value on the free boundary. This value is equal to the temperature of a crystallization. Also the law of change of the free boundary is known, it is called the Stefan condition. The Stefan problem with zero specific heat is the free boundary problem for an elliptic equation. Sometimes this model also called the Hele-Shaw problem (the problem models an incompressible viscous fluid between two parallel, narrowly placed plates).

The Stefan type problems for parabolic and elliptic equations arise in a lot of applications and have been studied extensively in the physics and applied mathematics literature using analytical methods of PDE and asymptotic analysis, numerical simulations, experiments. The theory of classical solvability for these problems with the smooth data is developed by E.T.Hanzava, A. Friedman, D.Kinderlehrer, B.V.Bazaliy. The L-p theory for the Stefan type problems have been constructed by V.A.Solonnikov, J.M.Elliot, J.R. Ockendon, A.Friedman, E. Di Benedeto. In all of the above cited works, the initial domain is assumed to be smooth and the fixed and free boundaries have empty intersection. J.R.King, A.A. Lacey, J.L. Vazquez constructed some asymptotical solutions of the Hele-Shaw problem when the initial free boundary was a plane corner.

In our work, we have studied the Stefan problem for the heat and Laplace equation in the case of nonsmooth initial data, for example, when initial free boundary has corner points. It has been constructed the appropriate weighted classes where unique solution of the mentioned problems exists locally in time. We obtained the sufficient condition on the angle in the neighborhood of corner point when the geometry of the free boundary does not change for a some positive time: a so-called 'waiting time' phenomena.

Here we apply the method which was suggested early by B.V. Bazaliy in the case of the Stefan problem for heat equation in the domain with smooth boundaries. The technique consists in the reduction of the free boundary problem to the nonlinear problem in a fixed domain for two unknown functions. For this purpose, it is used the transformation like Hanzava. The Frechet derivative is constructed for the obtained nonlinear system of the partial differential equations, and the one-to-one solvability of a corresponding linear system is shown for certain conditions. After this the initial problem is reduced to the fixed point problem for a certain nonlinear operator. On this route the main analytical difficulties are connected with the proof of the one-valued solvability for the nonclassical linear boundary value problem for elliptic or parabolic partial differential equation with variables coefficient in the domain with corner points.