Complex Analysis Outline 1

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1. We the field of complex numbers, $\mathbb{C}$, such that if $z \in \mathbb{C}$, then $z = a + ib, a, b \in \mathbb{R}$. And we discussed the properties of z.
2. Because $z = a + ib$, we could denote z to be in a plane, with coordinate (a, b). Thus complex numbers follows some of the rules of $\mathbb{R}^2$. For example, the triangle inequality.
3. Consider the polar representation of z. We define (r, θ), thus, we have $z = r\cos\theta + ir\sin\theta$. Thus, we define $cis\theta = \cos\theta + i\sin\theta$. And from there, we could define roots of unity, and we discovered a homomorphism from addition of θ to multiplication of z.
4. Next, we defined a line in $\mathbb{C}$. $L = \left\{ z : Im\left(\frac{z-a}{b}\right) = 0 \right\}$
5. We finally defined the metrics in $\mathbb{C}$. By stereographic projection, we could map the $\mathbb{C}$ to $\mathbb{R}^3$, and then we define metrics to be the distance in $\mathbb{R}^3$. Naturally, we could derive the metrics on $\mathbb{C}$ from there.